Fiscal Policy Rules and the Sustainability of Public Debt in Europe

Stefan Collignon

Abstract

In this paper, the sustainability of public debt is interpreted as the result of the interaction of fiscal policy with the economic environment, and not as a statistical concept as in most of the recent literature. If debt must not explode over time, policy makers have to respond to the changing conditions in the macroeconomic environment. This paper defines the conditions which will ensure compliance of fiscal policy with the intertemporal budget constraint in the context of Europe's fiscal policy rules. The empirical part of the paper reveals that European public debt is sustainable in this respect, but questions regarding liquidity remain.

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The risk of a default on Greek sovereign debt has thrown the Euro into its first serious crisis and raised the issue of debt sustainability in Europe. To this day, no one has proven that Greek public debt is unsustainable and this shows the conceptual difficulties of the issue. In academic literature the concept of sustainability refers to a dynamic equilibrium, which does not require any significant change in fiscal policy, and it implies long term financial stability, where markets provide funds to cover the borrowing requirements. Sustainability does not mean budgets have to be balanced at all times, provided temporary deviations from the sustainable rate are duly corrected. However, when deficits become excessive and debt explodes, a governments' solvency is threatened. Borrowers are considered solvent as long as they can repay their debt and interest out of future revenue. Economists have a clearly defined criterion for solvability, namely respect for the intertemporal budget constraint. However, uncertainty about the fulfillment of this condition can undermine the confidence market have about a government's solvability and, therefore, dry out the liquidity necessary for refinancing new or maturing debt. The rising risk of default due to a liquidity crisis may then force a government to change policy, even if it is solvent and its debt is fundamentally sustainable. This paper will discuss policy rules that ensure that governments remain solvent, despite significant variations in liquidity requirements, and it will therefore throw a new light on the issue of debt sustainability.

Solvency requires that governments respect the intertemporal budget constraint, and most economic literature employs statistical unit root and cointegration tests to examine if the observed data are consistent with this requirement. Few papers look at the policy arrangements that generate these data. However, Bohn (2007) has shown that time series tests are incapable of rejecting the assumption of sustainability, because the intertemporal budget constraint is satisfied if the time series of the relevant debt variable is stationary after any number of differencing operations. I, propose a different approach to assessing Europe's debt sustainability by looking at governments' fiscal policy reaction patterns within the given context of economic growth and interest rates. Rather than deriving criteria for judging sustainability from observed data, I suggest modeling the process that generates sustainable data, and then check if the data are consistent with this model.

Debt sustainability requires that deviations from the long run equilibrium are systematically corrected, which requires that fiscal behavior follows certain rules that can ensure this. Yet, important questions

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remain: how is the equilibrium defined and how much deviation is acceptable? Is there some economic logic to the definition of sustainability, or can it be arbitrarily defined by the political process? And what are appropriate fiscal rules? A policy-based model of debt sustainability must give answers to these questions.

Not all countries base their policies on explicit fiscal rules like the European Union, but most, if not all, follow at least some implicit rules. The US government appears to have followed for decades an implicit policy rule which can be described as an error correction process in levels.[2] The European Union has established two sets of explicit policy rules. The Excessive Deficit Procedure (EDP),[3] first formulated by the Maastricht Treaty,[4] is to ensure compliance with budgetary discipline on the basis of two criteria: (a) the ratio of the planned or actual government deficit to gross domestic product ought not to exceed the reference value of 3% and (b) the ratio of government debt to gross domestic product ought not to exceed the reference value of 60%. Deviations are permissible only if they are exceptional and temporary and the ratio remains close to the reference value. When the European Commission and the Council of Finance Ministers (ECOFIN) have formally recognized that a member state runs an excessive deficit, its government is under the obligation to bring "that situation to an end within a given period"; otherwise it has to pay fines. Nevertheless, there is still significant flexibility in the procedural application of the EDP. The Stability and Growth Pact (SGP)[5] has therefore defined a second and more restrictive set of policy rules with the aim to further strengthen the EDP by tightening procedural regulations and requesting member states to maintain cyclically adjusted (i.e. *structural*) budget positions "close to balance or in surplus." [6] These two sets of rules complement each other, although it is not clear, whether the binding constraint on European fiscal policy is the EDP or the SGP.

There is still a lively debate about the usefulness of Europe's fiscal rules. They have been criticized for being too tight and creating a pro-cyclical and low-growth bias for fiscal policy; they were also attacked for being too loose, because they did not prevent countries like Greece to accumulate excessive deficits. Both criticisms may lead to the conclusion that Europe's public debt is not sustainable. Although there is evidence for procyclicality and non-compliance, critiques of Europe's rules rarely address how these rules affect the sustainability of debt directly.

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[2] See Bohn, 2005 and my discussion below.
[3] See art. 104 in the Consolidated Version of the Treaty on European Union and of the Treaty Establishing the European Community (Lisbon Treaty, 2008).
[4] Now incorporated into the Lisbon Treaty art. 126.
[5] The Stability and Growth Pact is secondary legislation in the form of two Council Regulations (EC Council Regulation 1466/97 and 1467/97. These Council Regulations have been amended by the "reform" of the SGP in 2005 (see EC Council Regulations 1055/2005 and 1056/2005).
[6] The interpretation of the formulation "close-to-balance over the medium term" is now generally accepted to mean balanced structural budget positions. See European Commission, 2002 and ECOFIN, 2003

In theory, the Stability and Growth Pact should guarantee that public debt remains sustainable, while allowing the automatic stabilizers to stimulate the economy in a recession and consolidate public finances in a boom. The rule would support economic growth because it minimizes cyclical variations and keeps interest rates low. In reality, few member states ever achieved balanced structural budgets, so that fiscal policy has become pro-cyclically tight in recessions (see below). This asymmetry could indeed become a handicap for growth and would affect debt levels negatively. Nevertheless, a balanced structural budget is not a necessary condition for achieving debt sustainability, although it is, of course, sufficient. The problem with Greece was not that the rules were inappropriate but that the Karamanlis government deliberately circumvented them. Corrective action is now taken by his successor Papandreou, but critics argue that the consolidation is too harsh. In these policy discussions, it remains unclear how much fiscal adjustment is needed to ensure debt sustainability.

The rest of this paper will clarify how explicit fiscal rules policies can contribute – or not – to making public debt sustainable. It will first establish the necessary conditions for sustainable debt-GDP ratios in the context of European fiscal rules and then provide empirical evidence for the realization of these conditions. Our estimates indicate that Member states in the Euro Area are solvent in the sense that their debt ratios are not explosive, although their equilibrium steady states may rise to substantial levels, which could cause serious liquidity problems.

1. Theory

1.1. The Concept of Fiscal Sustainability.

No universally accepted definition for sustainable fiscal policy exists, but scholars agree that exploding debt is not sustainable. Budget policy is constrained by the need to finance the deficit. If it were possible to borrow without restraint and to finance the interest on debt by additional borrowing, any pattern of deficits would be sustainable. However, governments encounter limits to how much they can borrow. They face a present-value borrowing constraint; in other words, governments must intertemporally balance their budgets by setting the current market value of debt equal to the discounted sum of expected future surpluses. A violation of intertemporal budget balance would indicate that the fiscal policy cannot be sustained forever, because the value of debt would explode over time at a rate faster than the growth rate of the economy. A sustainable fiscal policy must respect the present-value borrowing constraint, causing thereby the discounted value of debt to go to 0 at the limit (Quintos, 1995). This is the common idea behind all modern models of debt sustainability.

In empirical work, unit root and cointegration tests are commonly employed to examine if time series are consistent with the intertemporal budget constraint (Hamilton and Flavin, 1986; Trehan and Walsh, 1988 and 1991; Wickens and Uctum, 1993; Quintos, 1995; Alfonso, 2005). However, Bohn (2007) has shown that such tests are incapable of rejecting sustainability. The reason being that the intertemporal budget constraint proves to be satisfied if either the debt series or the series of revenue and with-interest spending are integrated of arbitrarily high order, i.e., if they are stationary after an arbitrary sequence of differencing. Furthermore, revenues and spending do not have to be cointegrated. He concludes that rejections of low-order difference-stationarity and cointegration are consistent with the intertemporal budget constraint and therefore suggests error-correction-type policy reaction functions as more promising for understanding debt problems. Such error-correction-type policy reactions are embodied in Europe's fiscal policy rules. It is therefore useful to determine the conditions under which this mechanism ensures sustainable public debt.

In line with the literature, we start with the government's budget constraint relative to GDP:[7]

(1) $d_t = (1 + r_t - y_t)d_{t-1} - s_t$

or:

(1a) $\Delta d_t \equiv d_t - d_{t-1} = (r_t - y_t)d_{t-1} - s_t$

The increase in the debt/GDP ratio depends on the balance between the growth adjusted debt service $(r_t - y_t)d_{t-1}$ and the primary surplus. The debt ratio will increase indefinitely if the real interest rate r exceeds the growth rate y, unless the primary budget yields a surplus that is sufficient to compensate the rising debt service. We call the relation of real interest rates to growth the *economic environment* and the level of the primary surplus the *fiscal policy stance*. Over the short and medium term the economic environment is affected by the interactions of fiscal and monetary policy. For example, interest rate hikes or a tighter fiscal stance may reduce demand and therefore growth. In the long run, however, the economic environment could also be determined by the fiscal stance because interest rates may rise with debt levels. As debt rises, markets may fear that insolvent governments will default on their obligations and they will ask for higher liquidity premiums. Nevertheless for the sake of simplicity, one can ignore the interdependencies between fiscal policy and economic environment. However, contrary to most of the literature, I will not assume that the economic environment is stable or dynamically efficient.

[7] $r = i - \pi$, where π is the rate of inflation (GDP deflator). (*r*-*y*) is the growth-adjusted real interest rate, d_t the debt/GDP ratio at date *t*, s_t is the ratio of the primary surplus to GDP. y_t stands for the real growth rate of the economy in period *t.*

Ignoring stock-flow adjustments, the absolute debt level grows by the amount of the current deficit, which consists of the primary deficit plus nominal interest charges. Normalizing to GDP yields the deficit/GDP ratio

(2) $$def_{t+1} = \Delta d_{t+1} + (y+\pi)d_t = (r+\pi)d_t - s_{t+1}$$

The possible paths of public debt implied by the sequences of fiscal policies (primary surpluses s_t) and economic environments ($r_t - y_t$) are:

(3) $$d_{t+n} = \left(\prod_{k=0}^{n} [1+(r-y)_{t+k}] \right) \cdot d_{t-1} + \sum_{j=0}^{n} \left(\prod_{k=j+t}^{n} [1+(r-y)_{t+k}] \right) s_{t+j}$$

Imposing some simplifying and arbitrary conditions, notably by assuming the economic environment as given and constant, the accumulation of debt over several periods *t=1…n* leads to:

(3a) $$d_{t+n} = (1+r-y)^n d_t - \sum_{j=o}^{n} s_{t+j}(1+r-y)^{n-j}$$

which implies, the present value of public debt in period *t+n* is equal to the initial debt in period *t* minus the present value of all future primary surpluses:

(4) $$\frac{1}{(1+r-y)^{t+n}} d_{t+n} = d_t - \sum_{j=1}^{n} \frac{s_{t+j}}{(1+r-y)^j}$$

Fiscal policy satisfies the intertemporal budget constraint if there is a trajectory such that the present value of all expected future primary surpluses equals the initial debt, i. e.

(5) $$d_t = E\left(\sum_{j=o}^{\infty} \frac{s_{t+j}}{(1+r-y)^j} \right)$$

otherwise bond holders would expect that part of their financial claims will not be repaid, and they would refuse holding such claims. The equivalent of (5) is the transversality condition (see Bohn, 2005):

(6) $$\lim_{n\to\infty} \frac{1}{(1+r-y)^n} d_{t+n} = 0$$

The transversality condition (6) requires r>y, i.e. an economic environment of dynamic efficiency[8]. Most of the literature assumes stationarity with a positive mean for the interest rate, or here of the growth-adjusted interest rate, as it facilitates the conceptional transition from budget identity to budget constraint (Bohn, 2005, 2007). However, if $r<y$, the intertemporal budget condition would imply that the discount factor below 1 is compensated by future borrowing. This seems odd, as governments could run Ponzi schemes (i.e. borrow to pay for debt service) for centuries. Yet, it is also possible that r<y may simply lead to falling debt ratios. This leads to an intriguing situation, where the debt ratio effectively implodes, i.e. is the mirror image of exploding debt. Nevertheless, most of the literature assumes stationarity of interest rates, or here of the economic environment, with a positive mean, as it facilitates the conceptional transition from budget identity to budget constraint (Bohn, 2005, 2007).

Empirically, it is not clear that the assumption of a stable macroeconomic environment with r>y is always realized. Bohn (1995, 2005) documents that average economic growth in the US economy has usually exceeded the average interest charge over the last 200 years. The U.S. government has been able to rely on economic growth to keep its debt-GDP ratio from rising. In Europe, such long time series for interest and growth are not available, but Figure 4 below shows that in the European Union, too, growth rates often have exceeded after-tax interest rates over the last half century. These empirical observations are hard to reconcile with theory. One solution has been suggested by Abel et al. (1989) and Zilcha (1992), who have shown that in a stochastic economy, dynamic efficiency depends on the relation between the growth rate and the return on “risky” assets. If government debt is considered “safe”, economic growth rates may well exceed low interest on government debt, especially when after-tax bond returns are considered. In any case, with time-varying growth and interest rates, one cannot assume a fixed and given economic environment. Fiscal policy, economic growth and interest on government debt are stochastic variables and this fact has significant theoretical and empirical consequences.

First, in a stochastic setting, the transversality condition (6) requires a zero limit of future government debt discounted not at the going average bond rate, but at a rate that depends on the probability distribution of revenues and spending across states of nature (Bohn, 1995: 258). However, empirically it is difficult to estimate such a rate from the data of a single observed fiscal time series. Secondly, the stochastic model imposes almost no restrictions on the *average* level of primary deficits because the government can trade off primary deficits in some states of nature against surpluses in other states. Therefore, econometric tests for stationarity are ill suited to check for sustainability and it would be more useful to look at how governments behave *when pursuing their fiscal policy objectives* in an

[8] For a deterministic model of dynamic efficiency, also known as the Keynes-Ramsey rule, see Blanchard and Fischer, 1989.

uncertain world. Bohn illustrates this by giving the example of a government with a fixed target for the debt-GDP ratio: If the interest rate is below the growth rate, the debt ratio at the beginning of the next period will be below the target level. The government can then run a primary deficit in the following year. A surplus is only needed when the growth rate falls below the rate of return on government bonds. Thus, whether fiscal policy is sustainable or not depends on the sign of the fiscal policy reaction with respect to the target: if an increase in debt is followed by an increase in primary surpluses, debt is sustainable.

Bohn (1998) has estimated the US reaction function for the primary surplus in response to a marginal increase in government debt. He finds significant reaction coefficients of the order of 2.8 and 5.4 percent and he concludes that “this provides reliable information about sustainability, regardless of how interest rates and growth rates compared" (p. 960). He recognizes that "permanent primary deficits will lead to excessive debt accumulation in at least some ‘bad’ states of nature", noting, however, that “a strictly positive and at least linear response of primary surpluses to the debt ratio is sufficient for sustainability”. The question then is what drives the accumulation of debt and how long does it take to revert? The key is the fiscal policy reaction function.

1.2. Europe’s Fiscal Policy Rules

European fiscal rules require the correction of budget imbalances, once the reference values of 3 percent for the deficit and 60 percent for the debt level have been exceeded. Under the Stability and Growth Pact, a zero structural deficit complements the 3 percent rule. These reference values cannot be normatively justified because theory cannot explain which values are “best”.[9] All we can judge is whether they are mutually consistent. Furthermore, the European policy rules do not prescribe how rapidly the correction of imbalances has to be achieved; they leave this decision to member states, although the European Council can give binding recommendations for the size of the consolidation effort.

In our analysis, policy behavior determines which debt levels are sustainable because we assume that the macroeconomic environment is exogenously given, at least in the short run, so that the *debt sustainability requires a variation in the primary budget surplus*. The EDP does not demand governments to correct the excess over the reference value fully and immediately, but only “to bring that situation to an end within a given period” (Lisbon Treaty, TFEU art. 126.7).We therefore formulate the fiscal policy rule as the reaction function:

[9] For example Reinhart and Rogoff (2009) show that less developed countries often default at debt ratios well below 60%.

(7) $\Delta s_t = \alpha(def_{t-1} - z_1) + \beta(d_{t-1} - z_2)$

where Δs_t is the increase in the primary surplus in period t, which reflects the correction of an overshoot over target "in the year following its identification" (EC Council Regulation 1467/97, art. 4). z_1 and z_2 are the target reference values for the deficit and debt ratio and α and β are coefficients for the adjustment speed by which governments respond to the two policy objectives.[10] The larger the adjustment coefficients α and β, the larger is the consolidation effort in response to the deviation from the reference values. α describes the proportion by which a government corrects the excess of a deficit, β does the same for the debt-GDP ratio. If both coefficients are zero, the government does not respond at all. α and β describe two separate models of fiscal consolidation. Even if $\beta=0$, debt levels will fall, as long as α is positive and if $\alpha=1$, the excess deficit is fully corrected in the following period. For $\alpha>1$, the government will increase the primary surplus sufficiently to ensure that the deficit will stay below the reference value in the following period. For $0<\alpha<1$, fiscal adjustment is spread over several years. The "reformed" rules of the SGP of 2005 stipulate that a country should undertake a budget correction of 0.5 percent of GDP in the second year after an excessive deficit has been declared.[11] Thus, if a country were running a deficit of 5 percent, the adjustment coefficient implicit in the new rules would be $\alpha=0.25$. In general one would expect that β is smaller than α and closer to zero. Bohn's (1998, 2005) work for the U.S. has focused exclusively on β, but I will argue that α is theoretically more powerful and - at least in Europe - empirically more relevant as supported by the statistical evidence below. Given the European policy rules so defined, one can now consider the conditions that ensure fiscal sustainability and then check how persistent the deviations are from the steady state.

1.3. Conditions for Sustainable Fiscal Policy

Equations (1), (2) and (7) can be expressed as a system of two simultaneous linear difference equations, rewritten in continuous time form

(8) $$\begin{cases} d' = (r - y)d - s \\ s' = (\alpha(r + \pi) + \beta)d - \alpha s - z \end{cases}$$

[10] Note that the fiscal policy reaction function as formulated by (7) is symmetrical, which is consistent with the Stability and Growth Pact (balanced structural budgets), while it functions as an asymmetric ceiling in the Excessive Deficit Procedure. We will interpret it therefore as the limiting condition for sustainability; actual performance should in general be better, because a deficit below 3% will not necessarily cause higher deficits in the next period.

[11] EC Council Regulations 1055/2005 (art.2.b): "the Council shall request that the Member State achieves a minimum annual improvement of at least 0,5 % of GDP as a benchmark, in its cyclically adjusted balance net of one-off and temporary measures, in order to ensure the correction of the excessive deficit within the deadline set in the recommendation."

where the prime indicates the time derivative and the constant $z = \alpha z_1 + \beta z_2$. Solving the difference equations for d(t) and s(t) gives the time path for the debt ratio and the primary budget position. If these two time paths converge to a pair of inter-temporal equilibrium values, i.e. the steady state, the fiscal policy is sustainable regardless of the intermediary adjustment dynamics, because it is consistent with the intertemporal budget constraint.

1.3.1. The Steady State

The equilibrium values of (8) are given by the solution for the particular integrals:

(9)
$$\bar{d} = \frac{z}{\alpha(y+\pi)+\beta} = \frac{\alpha z_1 + \beta z_2}{\alpha(y+\pi)+\beta}$$
$$\bar{s} = \frac{(r-y)z}{\alpha(y+\pi)+\beta} = (r-y)\bar{d}$$

If public finances are sustainable, the debt-GDP ratio must converge to an equilibrium position that is determined by the nominal growth rate, the reference values and the adjustment parameters. The steady state primary surplus equals the growth-adjusted debt service for the steady state debt. Under conditions of dynamic efficiency, it needs to be positive and would grow in proportion to the steady state ratio. We assume that the reference values and the reaction coefficients are structural values determined by the political system, notably the fiscal rules in the Treaty, but given that interest and growth rates vary over time, the steady state debt ratio will not be constant.

Equation (9) reveals a number of interesting facts. First, if we set $\alpha=0$ and only focus on the debt response $\beta>0$, the equilibrium debt ratio becomes identical with the debt target. Given that the European Treaty fixes the debt ratio at 60% of GDP, equation (9) says that if European governments target the debt objective rather than on deficits, the debt ratio would be stationary with a constant mean. However, if the policy reaction function is formulated with respect to *increases* to public debt, the steady state is always the debt ratio realized in the previous period so that the equilibrium path is a random walk.

Secondly, for $\alpha>0$, the debt dynamics get more complex. If $\beta=0$, the steady state for debt reflects the ratio of the deficit target (z_1) to the nominal growth rate ($y+\pi$). This is the consistency measure of the Maastricht Treaty mentioned in public debates.[12] But if $\beta>0$, the steady state for debt is lower. Note

[12] For example, 3% for deficits and 60% for the debt ratio are mutually consistent with a nominal growth rate of 5%, i.e. 2% inflation and 3% real growth. However, if the growth potential of the economy is only 2.5% and inflation 1.5%, the debt ratio consistent with 3% deficit would be 75% or a deficit of 2.4% with 60% debt.

also, that if z_1 =0, as implied in the medium term rule of the SGP, and if *β=0,* the steady state debt ratio and primary surplus are both zero.[13]

Thirdly, the most relevant case is, however, when $\alpha z_1>0$ *and/or,* $\beta z_2 >0$. The steady state is then a function of the nominal growth rate. It will rise, when real growth and inflation fall, and come down when the nominal growth rate increases. Hence, if fiscal policy aims at low debt ratios in the long run, and provided short term policy reactions assure stability, governments must focus exclusively on high nominal GDP growth.

Fourthly, the steady state will increase when nominal growth is reduced, although it will not explode, even if nominal growth approaches zero, as long as $\beta>0$,. However, this does indicate a growing urgency to focus on debt targets when the economy becomes stagnant.[14]

Fifthly, rising steady states imply respect of the intertemporal budget constraint, hence define solvency, but they also indicate a rising need for liquidity. Paradoxically, if markets lose confidence that the government is able to raise the necessary funds, a government may be pushed into default despite being fundamentally solvent,. In that case, debt sustainability will depend not only on solvency, but also on debt management.

1.3.2. Stability Conditions

Next, one needs to determine the conditions under which debt and primary surplus ratios will converge to the steady state. Convergence is faster if it proceeds in a monotonic fashion, than if the time path oscillates periodically in its convergence to equilibrium. Art. 126.2 of the Treaty on the Functioning of the European Union, stipulates that the deficit ratio must decline "substantially and continuously" and reach a level close to the reference value. This can be interpreted as the requirement for monotonic convergence. The difference between monotonic and periodic convergence becomes politically relevant when the periodic case gives the impression of exploding debt ratios despite long run convergence to equilibrium. Thus, we will establish the conditions under which convergence occurs either periodically or monotonically.

Solving the dynamic system (8) for the homogenous part yields the following condition for sustainable financial policies (for the formal proof, see Annex 1):

[13] DeGrauwe (2005:241) has made this point about the debt ratio and the deficit, but it obviously extends to the primary budget balance, which becomes identical to the deficit when the debt is zero.
[14] Japan is the closest case with an average nominal growth rate between 1999 and 2008 of 0.05%. European policy reform proposals afer the Greek crisis have given more prominence to thedebt target.

Proposition 1. *Given the policy system (8) and assuming α=0, a sufficient condition for the debt ratio's convergence to the steady state is*

$$\beta > (r-y)^2 .$$

Proposition 2. *Given the policy system (8), and assuming β=0, a sufficient condition for public finances to be sustainable is*

$$\alpha > \max\left\{r-y, \left(\sqrt{r+\pi}-\sqrt{y+\pi}\right)^2\right\} = \alpha_{\min}$$

This implies:

(i) $\alpha_{\min} = r-y$ *is the sufficient condition if r>y*

(ii) $\alpha_{\min} = \left(\sqrt{r+\pi}-\sqrt{y+\pi}\right)^2$ *is the sufficient condition if r<y*

Proposition 3. *Weak and strong sustainability are defined by the convergence to the steady state. One can distinguish three cases depending on the value of* α *:*

(i) *The* ***critical*** *or aperiodic case defines the condition for switching between cyclical (pseudoperiodic) and monotonic convergence of the debt ratio's path to the intertemporal equilibrium:*

$$\alpha_{crit} = \left(\sqrt{r+\pi}+\sqrt{y+\pi}\right)^2 .$$

(ii) ***Weak sustainability*** *is defined as the periodic case, which exhibits damped and oscillating convergence if:*

$$\alpha_{\min} < \alpha < \alpha_{crit}$$

(iii) Strong sustainability *is defined as the monotonic case. The necessary and sufficient conditions for rapid convergence are:*

$$\alpha > \alpha_{crit}$$

These are elegant results with important policy implications. First, it is clear that the short term fiscal policy reaction function can ensure long term debt sustainability. Sustainable debt is the result of fiscal policy in the here-now that responds to the changing economic environment; there is no need for simplifying *ad hoc* assumptions regarding interest rates, long-term forecasts for future liabilities, intergenerational accounting, etc. The reason is simple: if the system converges to the steady state, the intertemporal budget constraint is always fulfilled, even in time-varying macroeconomic models, because the political reaction function generates the required increases in primary surpluses. Secondly, the results also show that *policy rules*, which specify actions to be taken in a particular environment, do not have to change under normal circumstances, although the *fiscal policy stance*, which determines the action, will have to change. Thirdly, only when alpha is close to $\alpha_{\min}$, it is necessary to tighten the *policy* (i.e. increase α) in order to generate the surpluses (the fiscal *stance*) necessary to meet the growth-adjusted debt service. This need to tighten policy could create a pro-cyclical bias when the growth slow-down is caused by a negative demand shock. However, if α exceeds $\alpha_{\min}$ by a significant safety margin, the year-by-year fiscal policy can accommodate shocks without sacrificing sustainability. Fourthly, it matters what interest rate we use. Because the government collects taxes from bond holders, one should use the after-tax interest rate. An increase in the tax rate will simultaneously increase revenue and lower the post-tax interest rate, while a cut in government spending will only affect the primary surplus. Thus, fiscal consolidation is more efficient for sustainability if it is tax-driven.

2. Empirical Evidence

The theoretical model has assigned a crucial role to the adjustment coefficients α and β in determining the level of steady state debt and the convergence of actual debt ratios to this equilibrium. We will now estimate these coefficients for a group of European Union member states. With these coefficients one can calculate the steady state debt ratios and test if actual time series have been converging to equilibrium. If past sustainable policy reaction patterns have not changed, this would be an indication that Europe's public debt is sustainable.

2.1. Estimating the Fiscal Policy Reaction Function

We estimate the policy reaction function (7) in the form

(10) $$\Delta s_t = c + \alpha\, def_{t-1} + \beta\, d_{t-1} + \varepsilon_t$$

Where the constant $c = -z = -(\alpha z_1 + \beta z_2)$ amalgamates policy objectives and reactions. High policy *coefficients* should yield large negative constants, and tighter policy *objectives* (lower z_1 and z_2) would raise c.

2.2.1. The Data

Regarding the appropriate policy variable, several interpretations are possible:

(i) Based on the discussion in Section 1, Δs could be interpreted as a variation of the *actual primary surplus* position in response to the EDP. However, variations in the actual primary surplus reflect variations in the business cycle and may, therefore, present a distorted picture of policy reactions.

(ii) The variation of the *cyclically adjusted primary surplus* (CAPS) is more suitable to reflect fiscal policy responses, as it more closely controlled by governments. In line with the common practice followed by the European Commission, this is the variable use here.

(iii) An assessment of the sustainability of public debt under the rules of the Excessive Deficit Procedure (EDP) should use the time series of the *actual budget deficits* as the appropriate deficit variable.

(iv) An assessment of the Stability and Growth Pact (SGP) must take cyclically adjusted values for government borrowing, because the Pact stipulates balanced *structural deficits*.

The European Commission AMECO Database 2009 was used for the variables described in Annex 2. The analysis covers 14 members of the old EU-15, excluding Luxemburg. Sufficient data for the new member states were not available. Our sample covers harmonized Eurostat data for the 31-year period 1978-2009. The introduction of ESA 1995 has caused changes in concepts, methods, definitions and classification of the European System of National Accounts. As a consequence, some recent time series are incomplete. Where needed, missing data were substituted by the pre-1995 data set, which was supplied by the European Commission.

The length of our time series is relatively short, especially regarding the Euro Area period. This is a handicap when testing for structural breaks. In order to increase the sample size and to augment the power of time series analysis, data were pooled in seemingly unrelated regressions (SUR). This methodology is also justified by the fact that the error terms are correlated (see Annex 3). We also test for panel unit roots.[15] The tests reject the null of either common or individual unit root processes, although the Hadri test also rejects the null of no unit root in any of the series in the panel (see Annex 4). On balance we accept the assumption of no unit root.

The growth-adjusted real interest rate uses the AMECO long term interest rates based on national government bonds and GDP growth rates. Deducting taxes on income and wealth (using the average rate for 1999-2005) modifies the after-tax growth-adjusted real interest rate sometimes significantly

[15] Using the routine provided for 5 such tests in E-views (E-views, 2004: 514-525). These procedures are more powerful than the standard multivariable time series procedures testing for unit roots (see Levin et al. 2002).

(see Figure 4 below). All the relevant rates vary substantially over time, although, with the exception of Finland, they have never exceeded 10 percent in the positive or negative range. Standard ADF tests reject unit roots for the (*r*-*y*)-series at the 5 percent level for the time period 1978-2008 except for Greece, Ireland and Italy. For these countries, the KPPS test also rejects stationarity (see Annex 5). As discussed above, a unit root process is counterintuitive to the theory of dynamic efficiency, which implies a stable positive mean,[16] but even if we reject the hypothesis of the growth-adjusted real interest rate evolving as a random walk, the usual ad hoc assumption of a dynamically efficient unchanging macroeconomic environment is clearly not realistic, as time variations in interest and growth rates are substantial.

2.1.2 Responsiveness to the Excessive Deficit Procedure (EDP)

Policy responses to the excessive deficit procedure are estimated following the methodology in Collignon and Mundschenk (1999). Given the relatively small sample of annual observations, a SUR (seemingly unrelated regressions) system for all 14 "old" EU member states and a smaller group of SUR10 for the Euro Area (excluding Luxemburg and Greece) were also estimated.[17] Including a time trend improved the statistical quality of the SUR models, although this variable was rarely significant at the individual country level. Results are shown in Table 1. In general, the SUR-14 model is most efficient, although the differences to the other models are minor.

[16] My own calculations for the United States, using Fed data annually from 1929 to 2009 show a stationary growth-adjusted real interest rate with a *negative* mean of -1.652.

[17] At the suggestion of one anonymous referee, I have also estimated regressions for the USA and Japan. Unfortunately they have not been statistically significant. The most probable interpretation is that these countries do not pursue fiscal rules of the European kind and more research is needed to understand the policy rules in the USA and Japan. The estimates were:

	OLS estimates of alpha and beta for USA and Japan					
	USA			Japan		
R-square	0.077			0.010		
	constant	alpha	beta	constant	alpha	beta
coefficient	-0.005	0.066	0.005	0.001	0.137	-0.006
p-value	0.829	0.645	0.873	0.829	0.163	0.331

Table 1- Policy Responce to excessive deficit procedure

Country	Constant			Alpha			Beta			R-square		
	OLS	SUR14	SUR10 (€-Area)	OLS	SUR14	SUR10 (€-Area)	OLS	SUR14	SUR10 (€-Area)	OLS	SUR14	SUR10 (€-Area)
Austria	-0.013	-0.018	-0.018	0.407	**0.370**	**0.354**	**0.004**	0.015	0.016	0.167	0.157	0.154
p-value	0.323	0.110	0.115	0.029	0.010	0.014	0.851	0.427	0.408			
Belgium	**-0.035**	**-0.036**	**-0.037**	**0.169**	**0.160**	**0.162**	0.024	**0.026**	**0.027**	0.288	0.287	0.287
p-value	0.036	0.011	0.010	0.009	0.004	0.004	0.096	0.041	0.038			
Denmark	-0.005	-0.010	-	0.134	0.076	-	0.009	0.018	-	0.147	0.133	-
p-value	0.611	0.213	-	0.151	0.274	-	0.637	0.210	-			
Finland	-0.002	0.002	-0.004	0.130	0.163	0.106	0.009	-0.002	0.013	0.163	0.154	0.161
p-value	0.814	0.716	0.602	0.139	0.007	0.111	0.665	0.882	0.465			
France	-0.004	-0.003	-0.005	**0.292**	**0.243**	**0.283**	-0.010	-0.010	-0.008	0.223	0.216	0.222
p-value	0.355	0.421	0.241	0.010	0.001	0.002	0.277	0.180	0.322			
Germany	-0.015	-0.011	**-0.015**	**0.697**	**0.739**	**0.742**	0.001	-0.010	-0.002	0.448	0.435	0.445
p-value	0.069	0.107	0.036	0.000	0.000	0.000	0.955	0.444	0.879			
Greece	-0.022	-0.019	-	0.358	0.355	-	-0.004	-0.007	-	0.272	0.270	-
p-value	0.085	0.057	-	0.004	0.000	-	0.745	0.515	-			
Ireland	**-0.021**	**-0.026**	**-0.024**	0.111	**0.139**	**0.132**	0.023	**0.029**	**0.025**	0.341	0.318	0.333
p-value	0.021	0.000	0.001	0.130	0.014	0.027	0.103	0.004	0.021			
Italy	**-0.032**	**-0.030**	**-0.032**	**0.204**	**0.241**	**0.230**	0.019	0.015	0.018	0.249	0.227	0.242
p-value	0.050	0.015	0.013	0.006	0.000	0.000	0.143	0.158	0.103			
Netherlands	0.001	0.006	0.004	**0.319**	**0.328**	**0.258**	-0.015	-0.024	-0.018	0.226	0.220	0.213
p-value	0.961	0.564	0.694	0.010	0.000	0.010	0.447	0.158	0.310			
Portugal	-0.040	**-0.050**	**-0.038**	**0.498**	**0.588**	**0.494**	0.030	0.038	0.025	0.304	0.294	0.304
p-value	0.062	0.001	0.028	0.002	0.000	0.000	0.353	0.103	0.339			
Spain	-0.013	**-0.014**	**-0.014**	**0.291**	**0.283**	**0.287**	0.009	0.011	0.010	0.272	0.272	0.272
p-value	0.078	0.020	0.028	0.006	0.000	0.001	0.569	0.410	0.468			
Sweden	-0.027	-0.022	-	0.101	0.090	-	0.046	0.038	-	0.175	0.171	-
p-value	0.174	0.129	-	0.303	0.223	-	0.207	0.160	-			
UK	**-0.056**	**-0.060**	-	0.149	0.119	-	**0.113**	**0.123**	-	0.376	0.373	-
p-value	0.002	0.000	-	0.146	0.094	-	0.005	0.000	-			

The R^2 values are relatively low, but p-values for alphas are acceptable in the 5% range (shown in bold) in all Euro Area member states, although not in the out-countries. The constant is statistically valid in 7 cases and has the correct sign in all countries except the Netherlands where it is close to zero. Beta is significant only in Belgium, Ireland and the UK. Thus, within Euro Area, *high debt levels* (above 100%) have been of concern only in Belgium, but not in Italy.

The policy response coefficients α for deficits have a mean of 28% for OLS and SUR14 and of 30.5% for the Euro Area sample, with a maximum in Germany (74%) and a minimum in Finland (10-16%) and Denmark (7-13%). The average debt level correction is 2% for the whole sample and 1.1% for the Euro Area, with a maximum of 12.3% in the UK. Hence, fiscal discipline is higher within Euroland than outside and this can be interpreted as evidence that the SGP, which applies only to euro member states, has imposed a harder budget constraint on European monetary union. Furthermore, it is clear that Euro-governments respond mainly to the deficit target, and not to the debt target of the Excessive Deficit Procedure (except Belgium which has brought debt levels down by 1/3 in the 15 years after Maastricht was signed), and yet, the speed by which member states react to excessive deficits varies considerably. The half-life of removing the excessive deficit varies from one year in Germany and less than 2 years in Austria, Greece and Portugal; it is close to the euro-average in Italy (3 years), but more than 4 years in Belgium and Finland; and in Ireland it is even 5-6 years.

2.1.3 Responsiveness to the Stability and Growth Pact (SGP)

An important criticism of Europe's fiscal rules is that their rigidity prevents anti-cyclical stabilization policies or even imposes pro-cyclicality (Bayoumi and Eichengreen, 1995; von Hagen and Eichengreen, 1996; Eichengreen and Wyplosz, 1998; Bean, 1998). Proponents in favour of the rules argue that although the Stability and Growth Pact strengthens fiscal discipline, it allows the automatic stabilizers to absorb the swings of the business cycle, which are under normal circumstances well within the 3% range of the EDP (Artis and Buti, 2000). The Pact has stipulated that governments must balance their budgets "over the medium term", hence it has set a permanent target of zero structural budget deficits. This has consequences for the estimation of α. From a SGP-perspective, the consolidation effort (i.e. the increase in the cyclically adjusted primary budget) has two separate components: the SGP-component, which responds to structural deficits as governments must consolidate as soon as the structural budget position turns negative, and the EDP-component, which responds to the cyclical part of the deficit. If member states start out with a structural budget in balance or surplus and let the automatic stabilizers work, the policy reaction would be zero, unless the cyclical deficit exceeds 3%. An actively Keynesian anti-cyclical policy would lower the cyclically adjusted primary surplus in response to a negative demand shock and raise it when the shock is positive. However, if a government's initial position is a structural deficit, the total consolidation efforts must be larger than a simple response to an excessive cyclical deficit. In this case, the structural alphas will be larger than the unconditional alphas of Table 1. Furthermore, if the structural deficits are close to the 3% target, the automatic stabilizers will quickly push the deficits above the excessive deficit reference value, so that member states have to consolidate pro-cyclically. These two effects could distort the value of the unconditional alpha. It is, therefore, useful to disentangle the two components. Of course, for the purpose of debt sustainability, it is the structural alpha that matters most.

In a new regression, I distinguish between the structural α_{str} that indicates the consolidation efforts to cyclically adjusted deficits, and the cyclical α_{cyc} that reflects variations in cyclical deficits. A structural α_{str} fulfilling the conditions in proposition 2 and 3 would guarantee a sustainable steady state debt position. The cyclical α_{cyc} takes a negative sign, when governments pursue anti-cyclical policies and a positive sign when they consolidate pro-cyclically, but they would not affect sustainability. We regress our policy variable on the cyclically adjusted deficit and the cyclical component. The estimated reaction function is:

$$(11) \qquad \Delta s = \alpha_{str.}(structuraldef) + \alpha_{cyc}(cyclicaldeficit) + \beta d - z$$

Table 2 gives the results. The structural deficit coefficients are higher than in the overall estimates in Table 1: the mean is now 40% overall and 34.2% for the Euro Area. The structural deficit coefficients

α_{str} are now significant for Denmark and the UK. Cyclical coefficients are only significant in Spain, Ireland and Sweden. The signs of the coefficients are mostly positive, showing a tendency toward pro-cyclical fiscal consolidation in all Euro Area member states with the exception of Austria[18] and Italy, while the non-Euro Area countries all pursue anti-cyclical policies. Thus, most Euro Area member states must have started out with negative structural budget positions and this fact has forced them to act procyclically. These estimates confirm the result from the previous section that the SGP restricts fiscal policy in the Euro Area more than in the out-countries; it reinforces fiscal discipline by restricting anti-cyclical stabilization policies. Hence, the evidence supports the criticism of the SGP mentioned above. The question is, does it affect economic growth in the long run. If the pro-cyclical bias slows down economic growth, it would affect the debt dynamics negatively and to see the consequences for the sustainability requires checking whether the conditions in propositions 1-3 are fulfilled. Because the alpha-estimates for the Excessive Deficit Procedure in Table 1 are generally lower than those for the more restrictive Stability and Growth Pact, a conservative assessment of debt sustainability should work with the values in Table 1.

Table 2: Policy response and business cycle

Country	Constant		Structural Alpha		Cyclical Alpha		Beta	
	SUR14	SUR10	SUR14	SUR10	SUR14	SUR10	SUR14	SUR10
Austria	**-0.025**	-0.020	**0.700**	**0.647**	-0.506	-0.534	0.012	0.006
	0.007	0.077	0.000	0.000	0.057	0.110	0.423	0.751
Belgium	**-0.038**	-0.032	**0.150**	**0.162**	0.416	0.307	**0.028**	0.022
	0.013	0.061	0.009	0.013	0.220	0.428	0.037	0.139
Denmark	-0.010	-	**0.268**	-	-0.308	-	0.014	-
	0.243	-	0.009	-	0.072	-	0.343	-
Finland	0.005	0.003	**0.288**	0.253	0.034	-0.020	-0.004	0.001
	0.479	0.714	0.009	0.111	0.807	0.921	0.818	0.980
France	-0.002	-0.004	**0.259**	**0.282**	0.287	0.316	-0.012	-0.010
	0.551	0.385	0.019	0.025	0.082	0.088	0.175	0.306
Germany	-0.041	-0.025	**0.763**	**0.680**	-0.138	0.234	0.037	0.016
	0.052	0.401	0.000	0.007	0.651	0.589	0.238	0.713
Greece	**-0.174**	-	**0.771**	-	0.110	-	0.127	-
	0.000	-	0.000	-	0.753	-	0.000	-
Ireland	0.000	-0.001	**0.370**	0.255	**0.746**	0.715	-0.014	-0.008
	0.995	0.923	0.000	0.110	0.008	0.069	0.397	0.748
Italy	**-0.045**	**-0.043**	**0.281**	**0.244**	-0.418	-0.276	**0.026**	0.027
	0.002	0.020	0.000	0.001	0.105	0.394	0.027	0.071
Netherlands	-0.006	-0.003	**0.425**	**0.401**	0.010	0.123	-0.010	-0.013
	0.529	0.785	0.000	0.004	0.956	0.575	0.545	0.522
Portugal	**-0.071**	**-0.057**	**0.657**	**0.588**	0.205	0.123	**0.071**	0.052
	0.000	0.013	0.000	0.000	0.338	0.660	0.005	0.127
Spain	0.057	0.034	0.013	-0.094	**2.216**	2.000	-0.109	-0.063
	0.160	0.532	0.962	0.811	0.005	0.053	0.150	0.532
Sweden	-0.044	-	**0.516**	-	**-0.937**	-	0.069	-
	0.155	-	0.000	-	0.025	-	0.183	-
UK	**-0.090**	-	**0.174**	-	-0.181	-	**0.192**	-
	0.000	-	0.020	-	0.388	-	0.000	-

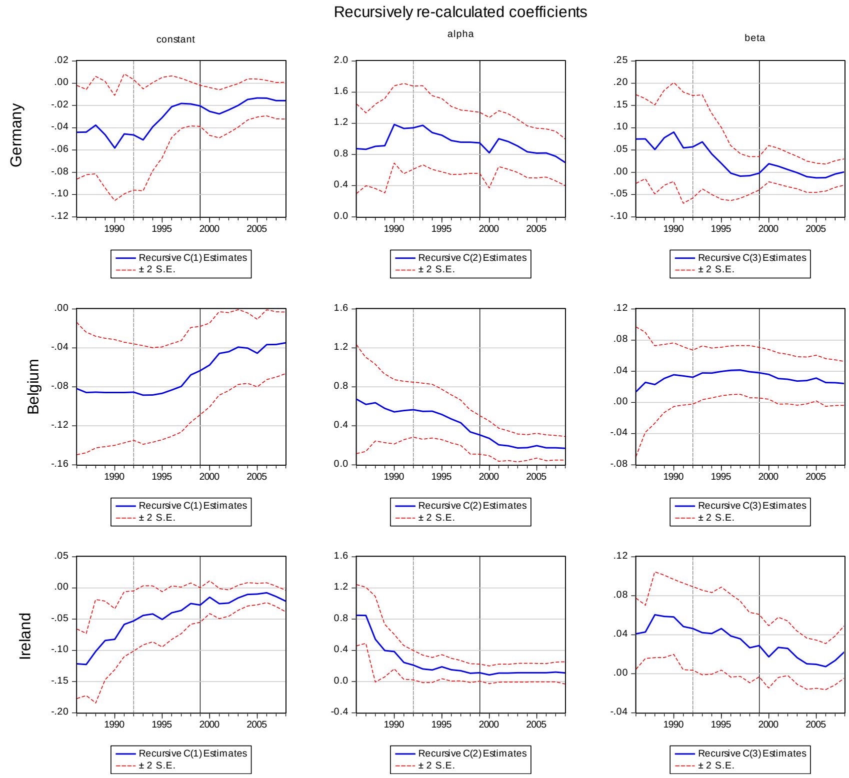
[18] This behavior may reflect the "long shadow of Austrokeynesianism" (Neck and Haber, 2005).

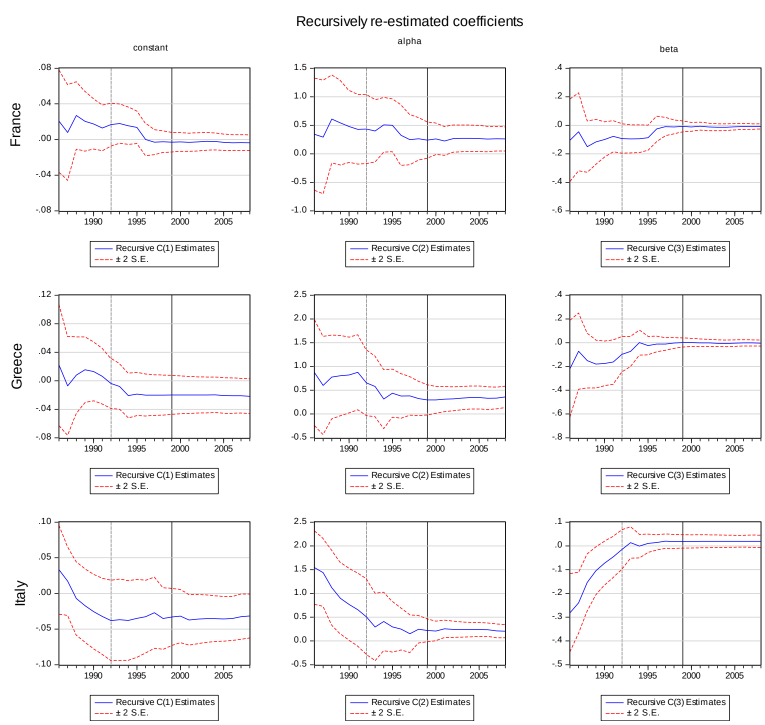
2.2. The euro as regime change?

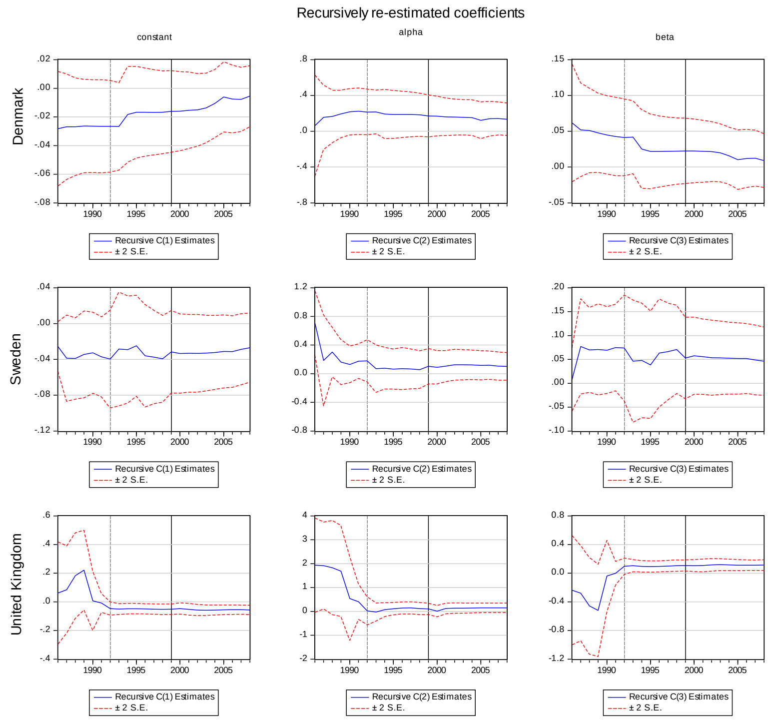
Given the small sample of observations, the reaction coefficients in Table 1 were estimated for the full 31-year period and the statistical power of the regressions was augmented by pooling the member states of the EU. However, one would expect that the introduction of the euro in 1999 and the accompanying fiscal framework have changed the policy behavior of governments. In this case, the policy reaction coefficient would not necessarily be stable. In order to test whether the new policy regime of monetary union has changed the adjustment behavior of member states, I first ran Chow tests for structural breaks imposing 1997 as the break year.[19] The hypothesis of no break in 1997 could not be rejected. At the request of one anonymous referee, I then checked for a break in 1992 with very ambivalent results. This led me to re-estimate recursively the OLS regressions. The results show parameter constancy for Austria, Finland, Netherlands, Portugal, Spain, Denmark and Sweden. In these countries, the commitment to the Maastricht Treaty did not change fiscal policy behavior. However, in the other member states, the coefficients did change, particularly during the transition period after the signing of the Maastricht Treaty, and the beginning of of Monetary Union. During this period, member states needed to meet the 3% objective and thereafter they were bound by the SGP. Figure 1 shows the dynamics. The first vertical line indicates the Maastricht Treaty, the second the start of European Monetary Union. The values for alpha are all reasonably stable over the first decade of the euro, but the constant changes sometimes significantly.

[19] The Maastricht Treaty was ratified in 1992 and the decision who would start EMU in 1999 was taken in 1997.

Figure 1.

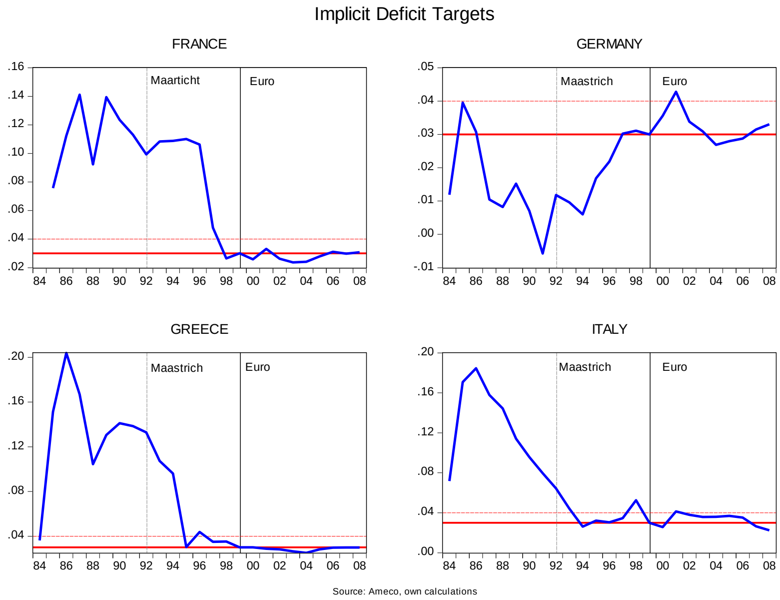

Recursively re-calculated coefficients
constant
alpha
beta
Germany
Belgium
Ireland
Recursive C(1) Estimates
± 2 S.E.
Recursive C(2) Estimates
± 2 S.E.
Recursive C(3) Estimates
± 2 S.E.


Recursively re-estimated coefficients
constant
alpha
beta
France
Greece
Italy
Recursive C(1) Estimates
± 2 S.E.
Recursive C(2) Estimates
± 2 S.E.
Recursive C(3) Estimates
± 2 S.E.



From equation (10) it is clear that a change in the constant will reflect changes in the reaction coefficients (α,β) or in the policy objectives (z_1, z_2). Given that most betas are not significantly different from zero, one can calculate the variation of the implicit deficit target. This is shown in Figure 2 for four member states, where beta could safely be ignored. The Maastricht Treaty led to a clear shift in deficit targets in Germany, France, Italy and Greece. In West-Germany the conservative government of the 1980s had ambitious consolidation objectives, which totally changed after unification. The chaotic policy shifts during the Schröder years, when fiscal consolidation was followed by tax cuts and then again by consolidation, show little concern for European commitments. Since the introduction of the euro, France and Greece have kept to the 3% objective or even undershot it, while Italy did not respect the deficit targets until the financial crisis hit it. The result for Greece is particularly puzzling, given its recent high deficits; either the data published by Eurostat are still distorted, or the excessive deficits in Greece were of a short term nature, caused by a slowdown in growth. It may, therefore, be more informative to analyze the debt dynamics in Greece. Steady state debt ratio in the next section provides some evidence that short term target deviation and economic growth shocks have dominated the Greek performance. However, the broad picture emerging form Figure 1 confirms constancy of alpha after 1999 in all member states except Germany. This allows a reasonably reliable assessment of Europe's debt dynamics.

Figure 2. Implicit deficit targets



2.3. Assessing the sustainability of Europe's public debt.

Following the theoretical argument made in the first part of this paper, public debt is sustainable if it does not explode, but converges to a steady state debt ratio. However, the steady state is not constant, but depends on the nominal growth rate of GDP and policy behavior, while the convergence depends on policy responses and the growth-adjusted real interest rate.

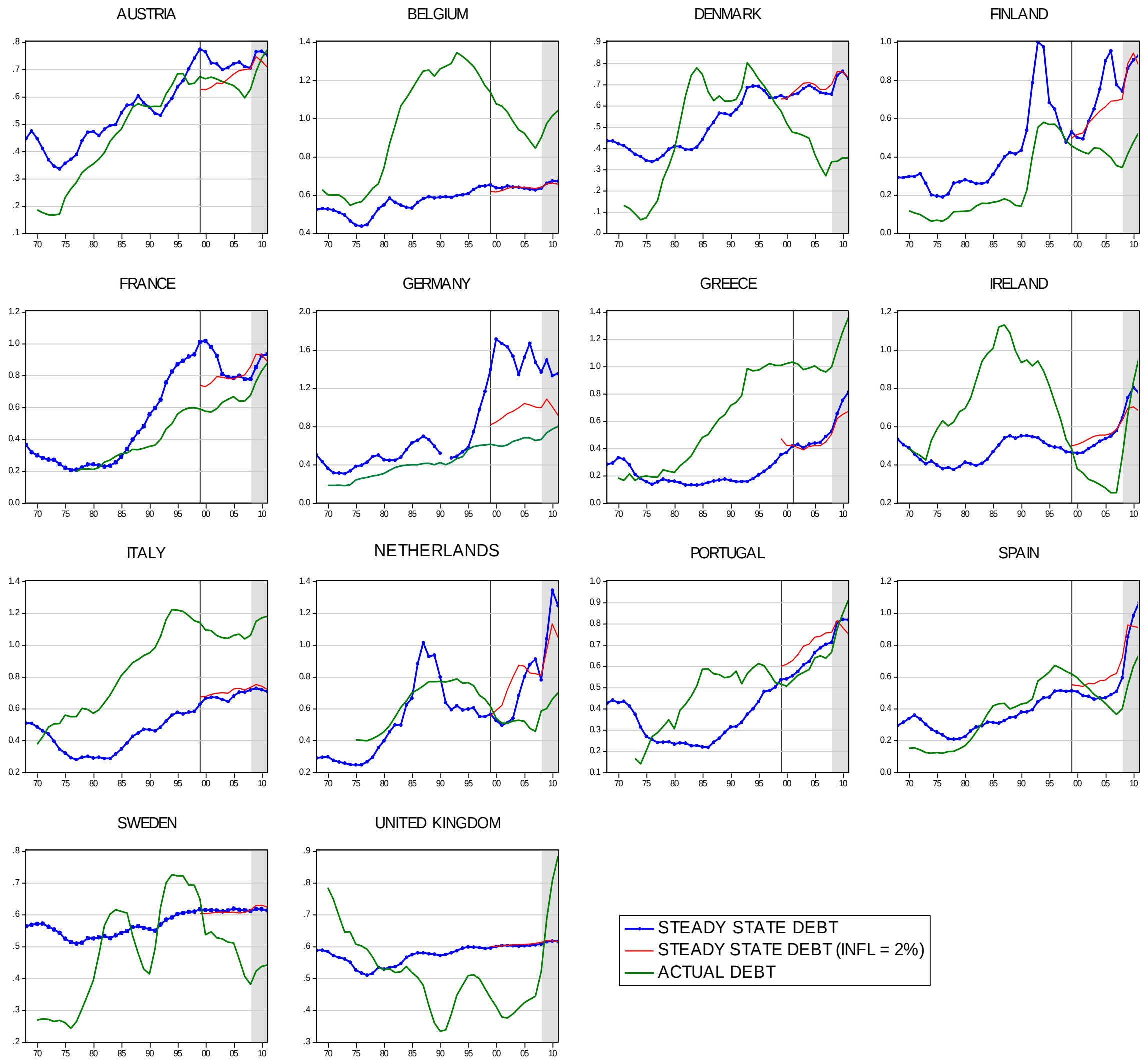
2.3.1.Calculating steady state debt ratios

Equipped with estimates for alpha and beta, one can calculate the steady state debt ratios. Figure 3 shows the historic evolution of the steady state debt ratios, as calculated by equation (9). They are constructed by taking the most significant coefficients of alpha and beta in table 1, but setting beta equal to zero if its value is negative. For growth rates, potential GDP was used, as published by the European Commission's AMECO data base (accessed in March 2010), including forecasts until 2011. To reduce data noise, the 4-year moving average of the GDP-deflator for inflation was given preference over annual rates. Because inflation is negatively affected by the depressed economic environment in the crisis, an alternative series, based on potential growth plus the ECB inflation target of 2 %, was calculated. Finally, figure 3 also shows the actual debt ratio.

In nearly all member states, the steady state debt level has risen during the disinflation period of the 1980s and 90s, when monetary policy was tight, interest rates were high and growth was slow. After the start of monetary union, it stabilized in Austria, Belgium, Greece and Spain; it came down in France and Germany, but it rose in Finland, Ireland, Italy, Netherlands and Portugal. However, the financial crisis has pushed up the steady state debt ratio in nearly all member states. The effect is particularly pronounced in the Southern European member states, which have suffered disproportionately from lower nominal growth. In some countries this is caused by very low inflation, far below the 2% target of the ECB, especially in Germany, Greece, Ireland and the Netherlands. If these member states realised at least 2 percent inflation, as measured by the GDP-deflator, the steady state debt ratio could have been significantly lower than it is today.

The actual debt ratios deviate from their steady state significantly, sometimes for very long periods. This is especially true for Italy, Belgium, Germany after unification, and to a lesser degree for France. Ireland, Sweden and possibly also Spain, France and Finland show a long n tendency for oscillating debt ratios. As section 2.1.2 has shown, the speed of policy reaction is very slow in some member states. From Figure 3 it is not intuitively obvious that actual debt levels are converging to the steady state. The next section will therefore formally test whether the stability condition is fulfilled.

Figure 3. Debt ratio steady states



2.3.2. Testing for convergence

Given that the practice of the European fiscal policy is mainly concerned with the deficit and not with the debt target, we can concentrate our analysis on proposition 2 and 3. A necessary condition for fiscal stability is a reaction coefficient $\alpha \geq \alpha_{\min} = r - y$ if *r*>*y*, and $\alpha \geq \alpha_{\min} = \left(\sqrt{r+\pi} - \sqrt{y+\pi}\right)^2$ if *y*>*r*. These conditions allow both, oscillating and monotone convergence to the steady state. Strong sustainability requires monotone convergence with the condition: $a > a_{cri} = \left(\sqrt{r+\pi} + \sqrt{y+\pi}\right)^2$. Before assessing which conditions are satisfied in the European case, a look at the growth-adjusted interest rate is required.

As discussed above, periods of dynamic inefficiency, when rapid growth exceeds long term interest rates, are more frequent than periods of dynamic efficiency. In the United States, this was true for the last 200 years (Bohn, 2005); for 1950-2008 the growth-adjusted interest rate was -1.1%.[20] In Europe, only in Austria, Belgium and Denmark have average interest rates been higher than growth rates over the 1961 to 2008 period. However, as it was discussed earlier in the paper, it would be more appropriate to use of after-tax interest rates to calculate the critical values.[21] Hence, the growth adjusted interest rate must be corrected by the tax share of nominal interest payments, which also depends on inflation. The after-tax growth-adjusted interest rate is $(1-\tau)\mathrm{i}-\mathrm{g}=(\mathrm{r}-\mathrm{y})-\tau(\mathrm{r}+\pi)$, where τ is the tax rate, π inflation and *i* and *g* are nominal interest and growth rates. Thus, an economy with high explicit tax rates on capital income or with high rates of inflation will reduce the need for fiscal adjustment; a deflationary economy ($\pi < 0$) with very low interest rates will increase it.

Figure 4 shows the growth-adjusted real interest rate before and after tax for European member states since the 1960s. Three clearly distinct “regimes” are perceptible: the Golden Age under the Bretton Woods system of the 1960s and early 70s, where growth rates have exceeded interest rates; the disinflation period of the 1980s and 90s, where interest rates were high relative to growth; and the period of European Monetary Union after 1999, when interest rates have generally been of similar size as growth rates. Thus, one would expect that debt ratios fell in the 60s and 70s, rose in the 80s and 90s and stabilized after 1999.

This is confirmed by Figure 3. The low growth-adjusted interest rates after 1999 reflect the improved macroeconomic environment with more accommodating monetary policy and higher growth during the euro’s first decade. However, 2007-9 financial crisis had made a very uneven impact in the different member states. Negative growth has pushed the growth-adjusted real interest rates up, often into the positive range. In Portugal and Ireland, it has seriously deteriorated the economic environment for sustainable debt, but in the future Europe’s lower growth potential [22] is likely to raise the growth-adjusted real interest rate permanently, unless it

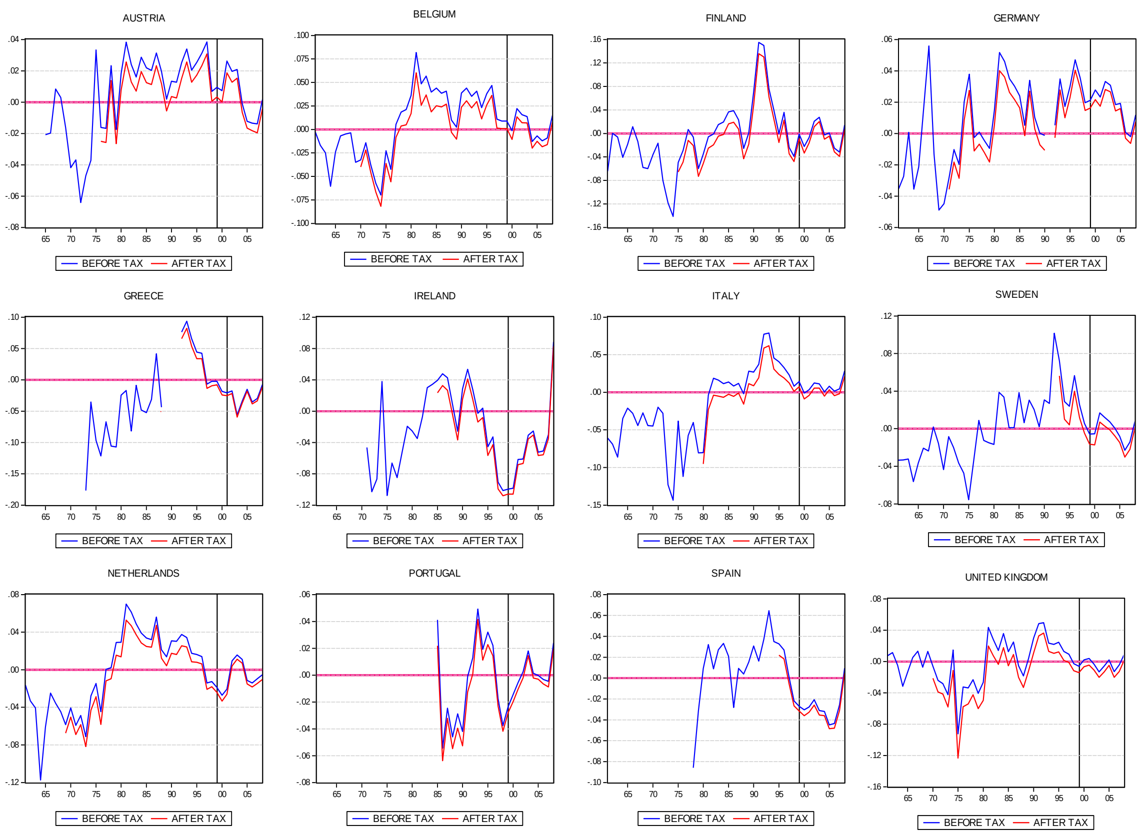
[20] Own calculations, based on Fed data.
[21] Strictly speaking, one should also correct it by the amount of debt held by the central bank under open market operations, but this argument could be ignored in Europe, as the ECB only rarely buys government debt.
[22] For a discussion of Europe’s potential growth after the crisis, see Commission, 2009.

is compensated by sustained low interest rates. Thus the level of the growth-adjusted interest rates, as reflected by these three regimes, is crucial for the analysis of debt convergence and one must take these different regimes into account when formulating fiscal policy stances. However, despite the common features under each of the three regimes, important differences between individual member states exist. The highest after-tax rate was observed in Finland in 1991 with 13.5%, the lowest in the UK in 1975 with -13.3%. After 1999, the *average* before-tax rate varied in a range of 1.8 in Germany and -4.2% in Ireland. Volatility was also highest in Ireland, representing Europe's minimum in 1999 at -10% and in 2008 the maximum at 8.8%. Thus, the critical values for debt sustainability vary significantly between member states and they may explain the different sizes of the adjustment gaps between observed and steady states in Figure 3.

Given that we have a non-negligible probability that the growth-adjusted interest rates follow a random walk,[23] the most appropriate procedure to compare (*r*-*y*) and α would be a test for co-integration of a time-varying α and looking at the co-integrating vector. However, with the small sample and relatively short time series for the Euro Area, this is not reasonable. Furthermore, section 2.2. has shown parameter constancy at least for the Euro Area. We therefore, with hesitation, ignore the unit root argument and compare the estimated α with the means of (*r*-*y*) for different time periods. We will distinguish between the high interest regime of the 1980s and 1990s, and the low interest period of European Monetary Union from 1997-2008.

[23] ADF tests do not reject unit roots for most, but not for all member states.

Figure 4. Growth-adjusted real interest rate



Source: AMECO

3.3.3.. Weak sustainability

Table 4 compares the estimated alphas with the *average required minimum* ($\alpha_{\min} = r - y$ and $\alpha_{aftertax} = (1-\tau)i - g$) provided r>y for the total time period of our regression and for the two sub-periods. If *r<y*, $\alpha_{\min} = \left(\sqrt{r+\pi} - \sqrt{y+\pi}\right)^2$ before tax or $\alpha_{\min} = \left(\sqrt{(1-\tau)(r+\pi)} - \sqrt{y+\pi}\right)^2$ after tax. Figures in italics indicate the critical values for *r<y*. It is immediately evident[24] that the sufficient condition for debt sustainability is fulfilled for all countries and all periods. This applies to the average values as well as for the maximum values observable in Figure 4. Hence, even if the SGP has made no difference to the *conduct* of fiscal policy, the *overall fiscal policy orientation* of Euro-member states is sustainable.

[24] Formal F-tests support the intuition, except in the case of Finland.

Table 4: Meeting the Sustainability Condition									
	1978-2008			**1978-1996**			**1997-2008**		
	α	α_{min}	$\alpha_{aftertax}$	α	α_{min}	$\alpha_{aftertax}$	α	α_{min}	$\alpha_{aftertax}$
Autstria	0.407	0.015	0.007	0.436	0.020	0.012	0.722	0.008	0.001
Belgium	0.169	0.024	0.013	0.201	0.037	0.023	1.063	0.004	*0.001*
Denmark	0.134	0.032	0.006	0.152	0.049	0.016	0.079	0.006	*0.001*
Finland	0.130	0.014	0.000	0.046	0.028	0.011	0.224	*0.002*	*0.004*
France	0.292	0.019	0.012	0.296	0.022	0.014	0.310	0.015	0.009
Germany	0.697	0.021	0.014	0.821	0.021	0.012	0.609	0.020	0.015
Greece	0.358	*0.030*	*0.026*	0.422	*0.083*	*0.077*	0.641	*0.000*	*0.000*
Ireland	0.111	*0.000*	*0.000*	0.078	0.004	*0.000*	0.415	*0.005*	*0.007*
Italy	0.204	0.012	*0.003*	0.256	0.013	*0.007*	0.437	0.010	0.003
Netherlands	0.319	0.017	0.008	0.295	0.032	0.021	0.607	*0.000*	*0.000*
Portugal	0.498	*0.010*	*0.007*	0.657	0.001	*0.015*	1.012	*0.000*	*0.000*
Spain	0.291	*0.004*	*0.002*	0.275	0.014	0.002	0.486	*0.000*	*0.001*
Sweden	0.101	0.016	*0.000*	0.092	0.025	0.003	0.159	0.001	*0.002*
United Kingdom	0.149	0.008	*0.000*	0.118	0.014	*0.002*	0.251	*0.000*	*0.002*

3.2.2. Strong Sustainability and Rapid Convergence to the Steady State

Strong sustainability implies rapid and monotonic convergence to the steady state. The critical value $a > a_{cri} = \left(\sqrt{r+\pi} + \sqrt{y+\pi}\right)^2$ indicates how fast a government must consolidate its deficit, if the fiscal variables are to converge straight without periodic oscillations to their inter-temporal equilibrium. This condition may be relevant for the assessment of changes in the debt ratio in the short-run. For if a country's debt were converging to the steady state, but only with oscillations, an increase in the observed debt ratio may be compatible with sustainability. This may confuse the judgment and could be seen as violating the Treaty provision of "approaching the reference value at a satisfactory pace" (art. 104c). Policy makers should then impose stronger policy responses, i.e. higher alphas.

Table 5: Test for Rapid Convergence									
	1978-2008			1978-1996			1997-2008		
	α	α_{cri}	α-αcri	α	α_{cri}	α-αcri	α	α_{cri}	α-αcri
Belgium	0.169	0.276	-0.107	0.201	0.327	-0.127	1.063	0.196	0.867
Portugal	0.498	0.326	0.172	0.657	0.455	0.202	1.012	0.157	0.855
Autstria	0.407	0.253	0.154	0.436	0.279	0.157	0.722	0.214	0.508
Netherlands	0.319	0.267	0.052	0.295	0.326	-0.030	0.607	0.176	0.431
Greece	0.358	0.286	0.072	0.422	0.300	0.122	0.641	0.221	0.420
Germany	0.697	0.246	0.451	0.821	0.274	0.547	0.609	0.206	0.403
Spain	0.291	0.277	0.013	0.275	0.342	-0.067	0.486	0.176	0.310
Italy	0.204	0.278	-0.073	0.256	0.355	-0.099	0.437	0.162	0.275
Ireland	0.111	0.334	-0.223	0.078	0.398	-0.320	0.415	0.231	0.184
France	0.292	0.266	0.026	0.296	0.329	-0.033	0.310	0.171	0.140
United Kingdom	0.149	0.257	-0.108	0.118	0.299	-0.182	0.251	0.191	0.061
Finland	0.130	0.303	-0.173	0.046	0.357	-0.311	0.224	0.218	0.006
Sweden	0.101	0.288	-0.187	0.092	0.336	-0.244	0.159	0.212	-0.053
Denmark	0.134	0.325	-0.191	0.152	0.422	-0.271	0.079	0.174	-0.095

Table 5 shows the relevant critical alphas $a > a_{cri} = \left(\sqrt{r+\pi} + \sqrt{y+\pi}\right)^2$ next to the actual policy coefficients. Given that the pre-tax rate is the tougher condition to meet, we only show the critical alphas for these interest rates. We find, first, that all Euro Area member states, including Greece, fulfill the condition of strong sustainability, i.e. monotonic convergence to the steady state. However, this is only marginally true for Finland and not the case for the two Scandinavian out-countries. Second, the change in economic environment following the creation of European Monetary Union has lowered the critical values everywhere, although less so in the non-Euro member states. Before monetary union started, i.e. during the high interest rate period 1978-1996, only Portugal, Austria, Greece and Germany fulfilled the conditions for strong convergence. Hence, monetary union has accelerated the convergence to the steady state, because of lower levels of interest rates and higher growth in the first decade of EMU. Outside the Euro Area, the conditions for strong sustainability were harder to meet. The average real interest rate in the Euro Area was 2.1 versus 2.4 in the three non-euro countries, while economic growth was 1.9 versus 1.7 in the two samples. Hence, it is not only the steady states that evolve very differently in the European Union, but convergence to these diverse equilibria is also very unequal.

Conclusion

In this paper, the sustainability of public debt was interpreted as the result of the interaction of fiscal policy with the economic environment, and not as a statistical concept as in most of the recent literature. If debt must not explode over time, policy makers have to respond to the changing conditions in their tax base (economic growth) and to the cost of finance (interest rates). Policy rules can help to ensure that at given moments the specific fiscal policy stances taken by governments are adjusted to changes in the environment so that debt will not explode, and this paper has defined the conditions which will ensure compliance with the intertemporal budget constraint. The empirical part of the paper has shown that European public debt is sustainable in this respect.

However, while compliance with the intertemporal budget constraint is a necessary condition for debt sustainability, it may not be sufficient. A government can be solvent in the sense that it can raise sufficient revenue in the future to pay for the debt service, but it may become illiquid if it cannot access financial markets at reasonable terms when old debt comes to maturity. A liquidity crisis can then turn into a solvency crisis, if high risk premia push interest rates up. The liquidity crisis of Greek sovereign debt in 2010 has brought this dilemma into the open.

Three lessons can be learned from this crisis based on the present analysis:
First, it may not be enough to respect the intertemporal budget constraint and to fulfill the sustainability criteria established in this paper, if financial markets, rightly or wrongly, fear the possibility of a sovereign default. This risk is likely to occur when steady state debt ratios start rising rapidly because growth has collapsed and prices are falling. Therefore, economic policy must focus on bringing nominal growth rates, i.e. real growth and inflation, up to reasonable levels, so that the steady state debt ratios will come down.

Secondly, it may be necessary to accelerate the convergence to the steady state in order to reassure markets that governments will comply with the intertemporal budget constraint. Thus, it is advisable to formulate minimum rules for the correction of an excessive deficit or excessive debt (i.e. the coefficients alpha and/or beta) that are significantly higher than the requirements derived from growth-adjusted interest rates in this paper. However, drastic fiscal consolidation must be tempered down if its effects reduce economic growth due to lack of demand. Fiscal policy must find a delicate optimum between insufficient consolidation which undermines financial markets' trust in fundamental solvency, and excessive consolidation which reduces growth. Such equilibrium cannot be established by *ex ante* rules. It is subject to the broad consideration of many policy parameters and this would genuinely be the kind of task of an economic government.

Thirdly, because financial integration is much deeper in a single currency area than between economies of different monies, the risk of contagion in a financial run is also much higher. If vanishing confidence in financial markets for one sovereign debtor causes a liquidity crisis that pushes up risk premia for other member states' debt, it may start to threaten the solvability of the whole region. There is therefore a case for collective action. The European Council has responded to the risk of systemic crisis in the Euro Area in May 2010, but the *European Financial Stability Facility* it created is a temporary *ad hoc* measure. In order to ensure its sustainability, monetary union will require a more permanent mechanism for ensuring that markets price the solvency risk of public debt and not liquidity defaults. The optimal answer to these problems is then the provision of large scale liquidity, rather than excessive budget consolidation. It is doubtful that the segregated national bond markets in Europe can provide the solution.

Problems with the liquidity and sustainability of public debt are not unique to Europe. They are inherently linked to the process of monetary integration. An early historical example is the United States. In 1790, Alexander Hamilton, the first Secretary of the Treasury of the United States, took the famous initiative, whereby the federal government would assume state debts incurred during the Revolution. It placed the country's most serious financial obligation in the hands of the federal, rather than the state governments. It encountered a lot of resistance by states, but it established the kind of clear and discernable reimbursement policy that inspired investors' trust that supported productivity and growth. Hamilton laid the foundations for the United States' economic future (Ellis, 2000). Europe has its Hamiltonian moment now.

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Annex 1. Proof of Proposition 1 and 2 (with Antoine Nebout)

For the proof, we solve the system of differential equations:

$$\begin{cases} d' = (r-y) - s \\ s' = (\alpha(r+\pi)+\beta)d - \alpha s - z \end{cases}$$

The characteristic equation of the homogeneous system is:

$$\lambda^2 + (\alpha - (r-y))\lambda + \alpha(y+\pi) + \beta = 0$$

We obtain the determinant of the second order equation:

$$\Delta = (\alpha - (r - y))^2 - 4(\alpha(y + \pi) + \beta)$$

The solutions of this second order equation are

$$\lambda_{1,2} = \underbrace{-\frac{\alpha - (r - y)}{2}}_{\text{h}} \pm \sqrt{\frac{\alpha - (r - y)^2}{4} - (\alpha(\text{y} + \pi) + \beta)}$$

The determinant will give us (i) the conditions of convergence with dampened oscillations (the pseudo-periodic case); (ii) the conditions for fast and monotonous convergence to the equilibrium; (iii) critical values for alpha establishing the minima conditions for stability and for monotonous convergence. We will take the economic environment *(r-y)* as exogenously given and focus on the policy reaction parameter *α* and *β*. To simplify the mathematics, we focus on the two polar cases (1) α=0, β>0 and (2) α>0, β=0. Empirically, the second case is more relevant, as β is statistically often not significant.

For the formal proof of this proposition, we solve the determinant of the homogenous system under three assumptions: (i) $\Delta < 0$, (ii) $\Delta > 0$, (iii) $\Delta = 0$.

(i). $\Delta < 0$ **: Pseudo-periodic case**

A sufficient and necessary condition on α et β for Δ to be negative implies:

$$\Delta < 0 \Leftrightarrow (\alpha - (r - y))^2 < 4(\alpha(y + \pi) + \beta$$

$$\Leftrightarrow \alpha(\alpha - 2(r + y - 2\pi)) - \beta < -(r - y)^2$$

Assuming β =0, which is justified on empirical grounds:

$$\Delta < 0 \Leftrightarrow f(\alpha) = \alpha^2 - 2\alpha A + B^2 \leq 0, \quad \text{with } A = r + y + 2\pi \text{ and } B = (r - y)^2$$

To determine the interval of the αs for which the parabolic function $f(\alpha) < 0$, we determine the two roots by computing the discriminator of $f(\alpha) = 0$:

$$\Delta = A^2 - B^2 = (A-B)(A+B) = 4(r+\pi)(y+\pi)$$

And we obtain the two characteristic roots:

$$\Rightarrow \begin{cases} \alpha_1 = r + y + 2\pi - 2\sqrt{(r+\pi)(y+\pi)} = (\sqrt{r+\pi} - \sqrt{y+\pi})^2 \\ \alpha_2 = r + y + 2\pi + 2\sqrt{(r+\pi)(y+\pi)} = (\sqrt{r+\pi} + \sqrt{y+\pi})^2 \end{cases}$$

Thus, the range of alphas, which are compatible with sustainability $f(\alpha) = \alpha^2 - 2\alpha A + B^2$ are such that the *sufficient and necessary condition* for Δ to be negative is: $\alpha \in [\alpha_1, \alpha_2]$.

A *necessary condition for the solution to be stable* is:.

Combining the two conditions we obtain:

If $\alpha_1 < r - y \Rightarrow \alpha \in [r - y, \alpha_2]$

If $\alpha_1 > r - y \Rightarrow \alpha \in [\alpha_1, \alpha_2]$

The first case is equivalent to:

$$r + y + 2\pi - 2\sqrt{(r+\pi)(y+\pi)} < r - y \Leftrightarrow 2(y+\pi) < 2\sqrt{(r+\pi)(y+\pi)} \Leftrightarrow y < r$$

So if $\begin{matrix} y < r \Rightarrow \alpha \in [r - y, \alpha_2] \\ y > r \Rightarrow \alpha \in [\alpha_1, \alpha_2] \end{matrix}$

In this case $\alpha \in [r\text{-}y, \alpha_2] \text{ or } [\alpha_1, \alpha_2]$.

Thus, either way, if $\alpha > r - y$, the system is converging to the steady state. If the system is dynamically efficient from the Keynes-Ramsey Golden Rule point of view, i.e. if $y < r$, then the lower bound for ensuring debt sustainability is $r - y$; if the system is dynamically inefficient in the Keynes-Ramsey sense ($y > r$), the lower bound is $\alpha_1 = (\sqrt{r+\pi} - \sqrt{y+\pi})^2 > r - y$, which means that high economic growth allows governments more leeway in consolidation.

The general solution of the system yields for the debt ratio:

$$d(t) = e^{ht}(m_1 \cos vt + i m_1 \sin vt) + \bar{d}$$

Where $h = -\frac{\alpha - (r-y)}{2}$ and $v = \sqrt{\frac{\alpha - (r-y)^2}{4} - (\alpha(y+\pi) + \beta)}$

The initial condition $d(0) = d_0$ gives us $m_1 = d_0 - \bar{d}$

If we take only the real part of the solution we get the time path:

$$d(t) = e^{ht}((d_0 - \bar{d})\cos vt) + \bar{d}$$

(ii). $\Delta > 0$: Monotonic convergence (if α > α_2)

For α **<** α_1 the system is unstable, and we will not examine this case.

In the case of $\Delta > 0$ and α **>** α_2**,** we have again the characteristic roots:

$$\lambda_{1,2} = \underbrace{-\frac{\alpha - (r-y)}{2}}_{h} \pm \sqrt{\frac{\alpha - (r-y)^2}{4} - (\alpha(y+\pi) + \beta)}$$

In this case the condition for convergence to the steady state is for both eigenvalues to be negative. Hence, a sufficient condition is:

$$\begin{cases} \alpha > r - y \\ \alpha(y+\pi) + \beta \geq 0 \end{cases}$$

And the general solution will be:

$$\begin{cases} d = m_1(e^{\lambda_1 t} + e^{\lambda_2 t}) + \bar{d} \\ s = n_1(e^{\lambda_1 t} + e^{\lambda_2 t}) + \bar{s} \end{cases}$$

The initial condition $d(0) = d_0$ gives us $m_1 = d_0 - \bar{d}$

Assuming $\lambda_1 < \lambda_2$ we obtain:

$$d(t) = \left[(d_0 - \bar{d})e^{(h-v)t}\left(1 + e^{2vt}\right)\right] + \bar{d}$$

Thus, if α **>** α_2 we have fast monotonous convergence to the steady state.

(iii). $\Delta = 0$**. Critical case (if** α **=** α_1 **or** α_2 **)**

If α **=** α_2 **,** the solution to the characteristic equation is:

$$\lambda = \lambda_1 = \lambda_2 = -\frac{\alpha - (r-y)}{2}$$

So a *sufficient condition* for steady state is:

$\alpha > r - y$.

And the form of the solution is:

$$\begin{cases} d = m_1(1+t)e^{\lambda t} + \bar{d} \\ s = n_1(1+t)e^{\lambda t} + \bar{s} \end{cases}$$

And the time path is: $d(t) = \left[(d_0 - \bar{d})(1+t)e^{ht}\right] + \bar{d}$

In analogy to physics, this case is named the aperiodic state and the convergence is quite fast – in any case much faster than in the periodic case.

Annex 2: Description of the variables

Name	Description	Source and code	Period covered*
Nominal GDP	Gross domestic product at current market prices	AMECO UVGD	1978 - 2008
Real GDP	Gross domestic product at constant market prices	AMECO OVGD	1978 - 2008
Potential GDP	Potential gross domestic product at 2000 market prices	AMECO OVGDP	1965 - 2008
Debt ratio	Gross public debt: General government consolidated gross debt:- Excessive deficit procedure (based on ESA 1995) and former definition	AMECO UDGGL	1978 - 2008
Actual deficit	Balances: Net lending (+) or net borrowing (-); general government:- Excessive deficit procedure Percentage of GDP at market prices (excessive deficit procedure)	AMECO UBLGE	1978 - 2008 GER: 1990 -2008
Structural deficit	Cyclically adjusted net lending (+) or net borrowing (-) of general government; Adjustment based on trend GDP. Percentage of GDP at market prices (excessive deficit procedure)	AMECO UBLGA	FIN: 1980 - 2008 GER: 1991 - 2008 GRE: 1988 -2008 IRE: 1985 - 2008 ITA: 1980 - 2008 SPA: 1995 - 2008 SWE: 1993 - 2008 UK: 1986 - 2008
Structural primary surplus (CAPS)	Cyclically adjustment of public finance variables based on POTENTIAL GDP; net lending (+) or net borrowing (-) excluding interest of general government adjusted for the cyclical component:- Adjustment based on potential GDP ESA 1995	AMECO UBLGBP	FIN: 1980 - 2008 GER: 1991 - 2008 GRE: 1988 -2008 IRE: 1985 - 2008 ITA: 1980 - 2008 SPA: 1995 - 2008 SWE: 1993 - 2008 UK: 1986 - 2008
Long term interest rate	Real long term interest rates, deflates GDO	AMECO ILRV	1978 - 2008
Tax rate	Current tax on income and wealth (direct tax): general government. ESA 95	AMECO UTYG	1978 - 2008
Inflation rate	Price deflator of gross domestic product at 2000 market prices	AMECO PVGD	1965 - 2008
Inflation rate	Inflation, average consumer prices, percentage change	IMF	1978 - 2008

Annex 3 : Variance-covariance matrix for 10 SUR countries

	Austria	Belgium	Finland	France	Germany	Ireland	Italy	Netherlan	Portugal	Spain
Austria	1									
Belgium	-0.0006	1								
Finland	0.2715	-0.1505	1							
France	0.1728	0.0437	-0.1895	1						
Germany	-0.0005	0.1547	0.3461	-0.1564	1					
Ireland	-0.3054	0.0196	0.2071	-0.0710	-0.1181	1				
Italy	-0.1364	0.1865	-0.2290	-0.1120	0.2478	0.0728	1			
Netherlan	-0.1425	-0.0715	0.1297	0.0171	-0.0045	0.3788	0.1965	1		
Portugal	-0.0756	0.0730	0.1143	-0.2967	0.0627	0.1464	0.3691	-0.2096	1	
Spain	0.1344	-0.1652	-0.1274	0.3077	-0.2528	0.3626	0.0261	-0.1883	0.0742	1

Annex 4: Group Unit Root Test Summary

TEST WITH INDIVIDUAL INTERCEPT

Sample: 1978 2008
Group unit root test: Summary
Series: D_AUT, D_BEL, D_DNK, D_FIN, D_FRA, D_GER, D_GRE, D_IRL, D_ITA, D_NED, D_POR, D_SPA, D_SWE, D_UKD, DEF_AUT, DEF_BEL, DEF_DNK, DEF_FIN, DEF_FRA, DEF_GER, DEF_GRE, DEF_IRL, DEF_ITA, DEF_NED, DEF_POR, DEF_SPA, DEF_SWE, DEF_UKD, SUKD, SSWE, SSPA, SPOR, SNED, SAUT, SBEL, SDNK, SFIN, SFRA, SGER, SGRE, SIRL, SITA
Exogenous variables: Individual effects
Automatic selection of maximum lags
Automatic selection of lags based on SIC: 0 to 3
Newey-West bandwidth selection using Bartlett kernel

Method	Statistic	Prob.**	Cross-sections
Null: Unit root (assumes common unit root process)			
Levin, Lin & Chu t*	-5.48355	0.000	42
Null: Unit root (assumes individual unit root process)			
Im, Pesaran and Shin W-stat	-11.5529	0.000	42
ADF - Fisher Chi-square	324.505	0.000	42
PP - Fisher Chi-square	307.479	0.000	42
Null Hypothesis: No unit root (assumes common unit root process)			
Hadri Z-stat	14.0235	0.000	42

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

TEST WITH INDIVIDUAL INTERCEPT AND TREND

Sample: 1978 2008
Group unit root test: Summary
Series: D_AUT, D_BEL, D_DNK, D_FIN, D_FRA, D_GER, D_GRE, D_IRL, D_ITA, D_NED, D_POR, D_SPA, D_SWE, D_UKD, DEF_AUT, DEF_BEL, DEF_DNK, DEF_FIN, DEF_FRA, DEF_GER, DEF_GRE, DEF_IRL, DEF_ITA, DEF_NED, DEF_POR, DEF_SPA, DEF_SWE, DEF_UKD, SUKD, SSWE, SSPA, SPOR, SNED, SAUT, SBEL, SDNK, SFIN, SFRA, SGER, SGRE, SIRL, SITA
Exogenous variables: Individual effects, individual linear trends
Automatic selection of maximum lags
Automatic selection of lags based on SIC: 0 to 3
Newey-West bandwidth selection using Bartlett kernel

Method	Statistic	Prob.**	Cross-sections
Null: Unit root (assumes common unit root process)			
Levin, Lin & Chu t*	-4.63542	0.000	42
Breitung t-stat	1.01577	0.845	42
Null: Unit root (assumes individual unit root process)			
Im, Pesaran and Shin W-stat	-8.98594	0.000	42
ADF - Fisher Chi-square	267.189	0.000	42
PP - Fisher Chi-square	335.863	0.000	42
Null Hypothesis: No unit root (assumes common unit root process)			
Hadri Z-stat	15.4221	0.000	

** Probabilities for Fisher tests are computed using an asymptotic Chi -square distribution. All other tests assume asymptotic normality.

Annex 5. Unit root tax for growth adjusted real interest rates (before Tax)

1) ADF-TEST: Null-hypothesis is (r-y) has a unit root

Country	t-stat	p-value	Exogenous	Sample
Austria	-3.094	0.038	constant, linear trend	1978-2008
Belgium	-4.067	0.017	constant, linear trend	1978-2008
Denmark	-3.908	0.024	constant	1978-2008
Finland	-3.321	0.023	constant	1978-2008
France	-2.817	0.068	constant	1978-2008
Germany	-2.989	0.049	constant	1981-2008
Greece	-1.106	0.682	constant	1978-1988 1992-2008
Ireland	-1.419	0.560	constant	1978-2008
Italy	-2.158	0.225	constant	1978-2008
Netherlan	-3.743	0.035	constant	1978-2008
Portugal	-3.084	0.043	constant	1985-2008
Spain	-3.110	0.037	constant	1978-2008
Sweden	-3.057	0.041	constant	1978-2008
UK	-2.910	0.056	constant	1978-2008

2) KPPS Test: Null series are stationary

Country	Exogenous			LM-Stat.
Greece	Constant	Kwiatkowski-Phillips-Schmidt-Shin test statist		0.182
		Asymptotic critical val	1% level	0.739
			5% level	0.463
			10% level	0.347
Ireland	Constant	Kwiatkowski-Phillips-Schmidt-Shin test statist		0.225
		Asymptotic critical val	1% level	0.739
			5% level	0.463
			10% level	0.347
Italy	Constant	Kwiatkowski-Phillips-Schmidt-Shin test statist		0.263
		Asymptotic critical val	1% level	0.739
			5% level	0.463
			10% level	0.347