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Proportionality, Disproportionality and Electoral Systems

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Different PR methods should be seen not as being more proportional or less proportional than each other but as embodying different ideas as to what maximizing proportionality means and, by extension, what minimizing disproportionality means. Each of the main methods of PR (d'Hondt, Sainte-Laguë, largest remainders) generates its own index of proportionality and, thus, its own way of measuring disproportionality. Applying these indices to competitive elections of the period 1979–89 shows a high correlation between the rankings produced by the various methods, but the ordering of countries is sufficiently different to require a choice to be made between the indices.

In any assessment of the merits of different electoral systems, the concept of proportionality always comes to the fore. Yet there is surprisingly little discussion of what exactly we mean by proportionality and how we should measure it. It is not always realized that different methods of PR (proportional representation), which may produce significantly different seat allocations for a given distribution of votes, should not be seen as inherently more or less proportional in their consequences. Rather, they produce different results because they embody different conceptions of what proportionality means and of what minimizing disproportionality entails. Each PR method minimizes disproportionality according to its own principles.

This paper will first review the main PR methods and discuss the principles underlying each of them, before tackling the question of the disproportionality produced by each. This entails a review of previous ideas as to how to measure disproportionality, with suggestions for new indices, and an application of the measures to recent competitive elections.

Methods of Proportional Representation

Only exceptionally might it be possible to distribute seats among parties in such a way as to produce perfect proportionality, defined here as a situation in which every party receives exactly the same share of the seats as it won of the votes. Otherwise, some deviation from perfect proportionality, that is to say some disproportionality, is inevitable. Every PR method will try to minimize the disproportionality created, that is, it will try to produce the outcome that is as close to perfect proportionality as possible. But once the notion of disproportionality is raised, we move away from an absolute standard to a relative one. Given two outcomes, neither of which

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represents perfect proportionality, how do we decide which is closer to perfect proportionality? How do we measure disproportionality?

Different ideas as to what we mean by minimizing proportionality have led to the invention of a number of methods of allocating seats to parties. We shall review six, five of which are, or could be, used to allocate seats to parties in the PR list systems adopted by most west European countries. Of these, the first five are known as highest average methods, as they involve dividing each party's vote total by the appropriate number from a predetermined sequence, depending on how many seats it has already been awarded, and awarding the next seat at each stage to whichever party presents the highest 'average'. The methods differ from one another in the numbers they use as divisors. Although the methods of operation of the various formulae are well known, the principles on which they are based are not, and so we shall look at the rationale for each method.

D'Hondt

The d'Hondt method is the most commonly used highest average method in Europe, being currently employed in Austria, Belgium, Finland, Greece, Iceland, Luxembourg, the Netherlands, Portugal, Spain and Switzerland for the allocation of some or all of the seats at parliamentary elections. It works as follows. Let v_i be the votes won, and s_i the seats received (so far), by the ith party. The d'Hondt formula then compares the respective values $v_i/(s_i + 1)$ for each party, and awards the next seat to whichever party can present the highest value. It thus employs the divisor sequence 1, 2, 3, 4 etc. Its operation is illustrated in Table 1, which relates to a constituency with five seats in which 100,000 votes are cast. Each party's vote total is divided by the first divisor, 1, and the first seat is awarded to the party whose 'average' is highest. This is party *A*. *A*'s vote total is now divided by the second divisor, 2, to give it its new current average, 30,000. This is higher than any other party's current average, so *A* is also awarded the second seat, and its vote total is divided by the next divisor in the sequence, 3, to give it its new current average, 20,000. *B*'s current average is now the highest, so it is awarded the third seat, reducing its average to 14,000. By a continuation of this process, party *A* will be awarded the fourth seat and then the fifth, obtaining a total of 4 seats, compared with 1 for *B* and none for *C*.

Given that *A* has won 60 per cent of the votes, why does d'Hondt not award it 3 seats, thus giving it its 'fair' share? The reason is that since some disproportionality is unavoidable in this case (neither *B* nor *C* can receive exactly its fair share of the seats), one party (or more) must be over-represented and one (or more) must be under-represented. The d'Hondt formula is concerned above all to minimize the over-representation of the most over-represented party. Giving *A* 4 seats (80 per cent) with 60 per cent of the seats means that *A*'s index of representation is 80/60, or 1.33. If, instead, *B* was awarded the fifth seat, its index of representation would be 40/28, or 1.43. If *C* received the fifth seat, its index of representation would be 20/12, or 1.67. Consequently, over-representing *A*, though not ideal, is in the d'Hondt formula's view less undesirable than over-representing either *B* or *C*.

The d'Hondt method's relatively severe treatment of tiny parties and its discouragement of party fragmentation have made it popular, at least with the dominant parties, as a practical formula for allocating seats to party lists, explaining its widespread deployment in western Europe. Because large parties tend to be over-

TABLE 1. Allocation of seats by d'Hondt highest average method

Party	Votes	Votes divided by first divisor (1)	Votes divided by second divisor (2)	Votes divided by third divisor (3)	Votes divided by fourth divisor (4)	Total seats
A	60,000	60,000(1)	30,000(2)	20,000(4)	15,000(5)	4
B	28,000	28,000(3)	14,000			1
C	12,000	12,000				0
Total	100,000					5

The numbers in brackets after the parties' vote totals indicate the award of a seat; party A was awarded the first seat and the second, party B the third, and so on.

represented under the d'Hondt formula, it is sometimes seen as the least proportional variant of PR (Lijphart, 1990: 484), though of course such a judgement involves certain assumptions about how disproportionality should be measured.

It has had many advocates, and Van Den Bergh (1956: 24) comments caustically that 'for decades many amateurs in this field rediscovered it every year'. This remark should be applied not least to d'Hondt himself, who 'invented' it nearly 90 years after Thomas Jefferson had done so (Balinski and Young, 1982: 18), and after it had been used in the United States for several decades to apportion seats in the House of Representatives to the various states. The method is known in the United States as the Jefferson method or the method of greatest divisors.

Sainte-Laguë and Modified Sainte-Laguë

Sainte-Laguë outlined his method in a short article in a French mathematical journal in 1910 (for an English translation, see Lijphart and Gibberd, 1977: 241–2). Unlike d'Hondt, he cannot be accused of plagiarism, for although his method proves to be identical to one devised by Daniel Webster in 1832, and used for several apportionments in the United States, his derivation of it was completely different from Webster's.

Sainte-Laguë employs the divisors 1, 3, 5, 7 etc; the nth divisor equals $2n-1$, and it awards the next seat to party A rather than party B if $v_A/(2s_A + 1)$ is greater than $v_B/(2s_B + 1)$. In the case illustrated in Table 1, it awards the seats on a 3-1-1 basis rather than the 4-1-0 outcome produced by d'Hondt. Compared with d'Hondt, it makes life harder for the large parties (in this case A), and consequently easier for small parties, by reducing more rapidly the 'averages' the former present; they are penalized more for every seat they have won.

The sequence 1, 3, 5, 7 etc has a sound mathematical basis rather than having been chosen for convenience; it derives from the way Sainte-Laguë viewed disproportionality and how it could be minimized (see his article and the section *Sainte-Laguë Index* below). Sainte-Laguë aimed to devise a method under which each voter would have, as far as possible, equal representation. Completely equal representation is achieved only when the fraction of a seat (or a deputy) 'owned' by a voter of the ith party (expressed as s_i/v_i) is identical to the overall seats:votes ratio.

The Sainte-Laguë formula, whatever its theoretical merits, proved too benign (or merely fair) to small parties for the liking of governing parties, so it is no longer used in Europe in its pure form. Instead, Denmark, Norway and Sweden use a modified version for awarding seats to party lists, under which small parties find it harder to win a seat. The sequence of divisors is the same as in the pure Sainte-Laguë system, except that the first divisor is not 1 but 1.4. Whereas d'Hondt awards the fifth seat to the largest party in Table 1, and Sainte-Laguë gives it to the smallest, the middle party receives it under the modified Sainte-Laguë method.

Equal Proportions and Modified Equal Proportions

The first country to tackle the question of what terms like 'proportionality' and 'disproportionality' mean was not, as is often thought, a western European country employing a PR formula to allocate seats at elections, but the United States. The American Constitution states that 'Representatives shall be apportioned among the several states according to their respective numbers'. The finite size of the House of Representatives meant that from the time of the first census in 1791, American legislators were confronted with the dilemma of how to operationalize the concept of proportional allocation of Representatives to states according to their population. Throughout the nineteenth century the d'Hondt and Sainte-Laguë formulae were the most frequently used, but in 1940 the approach now known as 'equal proportions' was adopted (for a history of apportionment in the United States, see Balinski and Young, 1982; Schmeckebier, 1976).

The equal proportions method was devised by E.V. Huntington, who pointed out the flaws in an earlier method devised by Joseph Hill (Huntington, 1921: 868) and devised in its stead one that correctly put Hill's principle into practice. Huntington argued that the Sainte-Laguë formula, then being championed in the United States under the name 'method of major fractions' by W.F. Willcox, did not really ensure that every citizen's fractional share of a Representative was as nearly equal as possible, as Willcox (and Sainte-Laguë before him) claimed. This is true, he observed, only if equality was measured in terms of the ***absolute*** difference between two quantities rather than the ***relative*** difference (the absolute difference divided by the smaller number). He argued that the latter was the true measure of difference: 'the inequality between 2 and 3 is 50 per cent, while the inequality between 100 and 105 is 5 per cent. Hence 100 and 105 are more "nearly equal" than 2 and 3 are' (Huntington, 1921: 861). The equal proportions method would ensure that each citizen's share of a member of the House of Representatives would be as nearly equal as possible by the criterion of the relative difference.

This was not all. It would also ensure that the number of people per Representative for each state (in a European context, the number of votes per seat for each party) would be as nearly equal as possible, again measured by the same criterion (the Sainte-Laguë formula does not achieve this result by any criterion). In fact, whatever type of equality was considered, Huntington's method was sure to maximize it, provided it was measured in terms of ratios rather than in terms of absolute differences.

The divisors that give effect to this approach are the square roots of successive pairs of numbers: $\sqrt{0 \times 1}$, $\sqrt{1 \times 2}$, $\sqrt{2 \times 3}$, $\sqrt{3 \times 4}$, etc. The first five divisors are thus 0, 1.41, 2.45, 3.46, 4.47. Despite its apparent theoretical soundness, Balinski and Young conclude that if it had been used to apportion Representatives to American states throughout the period 1790–1970, the smaller states would have received 3.4 per cent more than they would have received had all outcomes been perfectly proportional. The Sainte-Laguë formula would have produced more proportional results, with the smaller states over-represented by only 0.3 per cent (Balinski and Young, 1982: 73–6).

The equal proportions method has received little attention outside the United States, and is used nowhere else to allocate seats. The reason is not hard to see. Its first divisor of 0, if applied literally, means that every party presents an initial 'average' of infinity, so that even the smallest party will win a seat before any party will qualify for two. This poses no great problem in the United States, since under the Constitution every state is entitled to a minimum of one seat in the House, and apportionment uses the series of divisors *after the first* to allocate seats from the 51st onwards. But since parties, unlike states, can fragment themselves endlessly to gain a partisan advantage, the equal proportions method has generally been dismissed as of little relevance to the task of allocating seats to parties.

However, this is not necessarily the case; with suitable modifications it could have a part to play, if we were to replace 0 as the first divisor by a positive value. For example, if we wished to make it as hard for small parties to win a first seat under our 'modified equal proportions' system as is the case under modified Sainte-Laguë, we would need to make the gap between the first two divisors the same as in the modified Sainte-Laguë method, that is 3/1.4. We would thus use a first divisor whose value equalled $\sqrt{1 \times 2} \times 1.4)/3$, or 0.66, and the first five terms in the sequence would be 0.66, 1.41, 2.45, 3.46, 4.47. Rescaling this to a base of 1 gives the sequence 1, 2.14, 3.71, 5.25, 6.78, compared with the modified Sainte-Laguë sequence of 1, 2.14, 3.57, 5.00, 6.43. It would therefore be slightly more benign than modified Sainte-Laguë towards small parties, but would present almost the same threshold for the first seat in any given case.

Largest Remainders

The last method we shall discuss is not a divisor method. Instead, it works by employing a fixed quota, and the allocation of seats revolves around this figure.

The formula known in Europe simply as 'largest remainders' is perhaps the most straightforward of all methods of allocating seats; in the United States it is sometimes called the Hamilton method, as Alexander Hamilton proposed its adoption in 1792 for apportioning Representatives. It is used in Austria, Belgium, Denmark, Germany, Greece, Iceland and Italy for the allocation of some seats.

The quota it employs is calculated by dividing the total number of votes by the number of seats. The resulting figure is widely known in Europe as the Hare quota, and is also sometimes termed the natural, true or fair shares quota. After every party

TABLE 2. Allocation of seats by largest remainder (Hamilton) method

There are 100,000 votes and 5 seats, so the quota is 100,000/5 = 20,000

Party	Votes	Full quotas (first stage seats)	Remainder	Further seats	Total seats
A	60,000	3	0	0	3
B	28,000	1	8,000	0	1
C	12,000	0	12,000	1	1
Total	100,000	4	20,000	1	5

has received as many seats as it has Hare quotas, there will always be some unallocated seats, unless it happens that every party's 'fair share' comes to an exact number of Hare quotas, in which case perfect proportionality is attained. Except in this unlikely event, the unallocated seats are awarded to those parties whose unused shares of a quota, that is, remainders, are largest.

The operation of largest remainders is shown in Table 2. At the first stage, four seats are awarded, leaving just one to be allocated at the second stage. Since party *C*'s remainder is largest, the seat goes to *C*. Like Sainte-Laguë, it produces a 3-1-1 outcome, illustrating its kinder attitude to small parties than the d'Hondt formula.

Measures of Disproportionality

Reviewing the electoral systems literature, Lijphart (1985: 11) noted that the absence of any serious attempt to solve the problem of how to measure disproportionality was a clear sign of the low level of development of research in this field. This question has received surprisingly little discussion, certainly as far as multi-party competition is concerned (for discussion of disproportionality in contests involving only two parties in single-member constituencies, see Grofman, 1983). There are, of course, many ways of measuring inequality (Allison, 1978), but once these exceed a certain level of complexity the results they produce become difficult to interpret intuitively.

A key point that is often overlooked is that measures of disproportionality and allocation formulae are inextricably bound up with each other. Every method of seat allocation generates its own measure of disproportionality, and many measures of disproportionality implicitly endorse a method of seat allocation. The common procedure of testing the proportionality of various formulae by measuring their performance according to a criterion that in effect embodies one of the formulae being tested is thus methodologically questionable, if ultimately unavoidable.

There are two broad categories of measure, corresponding to the two types of allocation method that we have just discussed. Those in the first category concentrate on the absolute difference between a party's seats and votes, as does the largest remainders method. Measures in the second category focus on the ratio between a party's seats and its votes; most of these measures embody the values of one or other of the highest average methods.

Loosemore-Hanby Index

The first measure is based on vote–seat differences pure and simple. Calculating the overall disproportionality produced by an election entails adding the absolute values of the vote–seat differences for every party and dividing the result by 2 (for an illustration, see Table 3). This measure was devised (as far as the field of electoral systems research is concerned) by Loosemore and Hanby (1971: 469), and has now become widely known as their index. Mackie and Rose (1982 and 1991) subtract it from 100 and call the result the index of proportionality. Taagepera and Shugart (1989: 260–3) acknowledge that it has shortcomings but none the less prefer it to the alternatives. Many modifications have been proposed, but each seems to have some drawbacks when compared with the straightforward Loosemore-Hanby index, and Lijphart (1990: 483) notes approvingly that the latter has become the most widely used measure of disproportionality.

TABLE 3. Disproportionality at Luxembourg general election, 1979, as measured by six indices

Party	Votes %	Seats %	Loosemore-Hanby	Rae	Least squares	Sainte-Laguë	D'Hondt	D'Hondt (5%)
PCS/CSV	36.4	40.7	4.3	4.3	18.49	0.51	1.12	1.12
POSL/LSAP	22.5	23.7	1.2	1.2	1.44	0.06	1.05	1.05
PD/DP	21.9	25.4	3.5	3.5	12.25	0.56	1.16	1.16
PSD/SDP	6.4	3.4	3.0	3.0	9.00	1.41	0.53	0.53
PCL/KPL	4.9	3.4	1.5	1.5	2.25	0.46	0.69	0.69
EF	4.6	1.7	2.9	2.9	8.41	1.83	0.37	
SI	2.0	1.7	0.3	0.3	0.09	0.04	0.85	
AL	0.9	0	0.9	0.9	0.81	0.90	0	
Others	0.4	0	0.4		0.16	0.40	0	
Total			18.0	17.6	52.90	6.17		
Value of index			9.0	2.2	5.14	6.17	1.16	1.16

Notes on indices:
Loosemore-Hanby: sum |votes − seats| for each party; divide total by 2.
Rae: sum |votes − seats| for each party, excluding 'Others' and parties winning fewer than 0.5 per cent of the votes; divide total by number of included parties (in this case 8).
Least squares: sum (votes − seats)^2 for each party; divide total by 2; take square root of this value.
Sainte-Laguë: calculate (seats − votes)^2/votes for each party; sum these values.
D'Hondt: calculate (seats/votes) for each party; take largest such value.
D'Hondt (5 per cent): calculate (seats/votes) for each party with at least 5 per cent of the votes; take largest such value.

Source for votes and seats, Mackie and Rose (1991, pp.307–9).

What is not always appreciated is that this index is always minimized by the largest remainders method; the two are simply two sides of the same coin (for proof, see Appendix). Loosemore and Hanby's conclusion (1971: 475) that, when measured by their index, disproportionality is lowest in largest remainder systems is thus not very surprising. Consequently, using this index not only pre-judges what is ostensibly to be discovered, but it is also subject to all the anomalies and paradoxes of largest remainders, of which there are several.

The Loosemore-Hanby index's vulnerability to paradoxes can be illustrated by a case involving what is known in the United States as the 'new state' paradox, in which an allocation between two states is disturbed by the arrival of a newcomer. Suppose that there are 90 votes, 2 seats and two parties, *A* with 68 votes and *B* with 22 votes. Both largest remainders and Sainte-Laguë will award both seats to *A*. But if a third party now joins the fray and wins 10 previously uncast votes, so that the distribution is 68-22-10, the Hare quota rises from 45 to 50. Consequently, *A*'s remainder drops from 23 to 18, below *B*'s remainder of 22, so *A* and *B* now receive one seat each, even though the relationship between them has not altered at all. Sainte-Laguë, of course, as a divisor system, still awards both to *A*. The problem is that the Loosemore-Hanby index always by definition slavishly follows the largest remainders method, and so in this case it indicates that 2-0 is the least disproportional allocation when there are 90 votes but 1-1 is least disproportional when there are 100 votes. The index could be used to 'prove' that largest remainders delivers a more proportional result than Sainte-Laguë in the 100-vote case, even though the divergence between them arises only because largest remainders is vulnerable to paradoxes from which Sainte-Laguë is immune.

The Loosemore-Hanby index has the same merits and demerits as the largest remainders formula on which it is based. Its method is straightforward and easy to understand, but it is weakened by its vulnerability to paradoxes. These and other doubts have led to the development of other difference-based indices, to which we now turn.

Rae and Least Squares Indices

As noted above, many modified versions of the Loosemore-Hanby index have been proposed. The best known is that devised by Rae (1971: 84), which consists of adding the vote–seat differences of each party winning more than 0.5 per cent of the votes and then dividing the total not by 2, as Loosemore and Hanby advocate, but by the number of parties this criterion produces (see Table 3). The two indices are in fact trying to measure different things. Whereas the Loosemore-Hanby index measures the total disproportionality per *election*, the Rae index measures the total disproportionality per *party*. As an overall measure of disproportionality it is flawed, since a plethora of small parties each of whose vote totals just exceeds Rae's cutoff point will bring down its value to an artificially low level (see Lijphart, 1985: 10). It therefore overstates the proportionality of multi-party systems.

Nevertheless, there is a good idea behind Rae's proposal. Its rationale is that the sum of the vote–seat differences is not, as Loosemore and Hanby maintain, enough on its own to convey reliable information on the proportionality of an election outcome. We want to know something more about how this sum was reached. Does it derive from many parties each having a small vote–seat difference or from a few each having a large difference?

Consider two elections. In one, there are just two parties: one wins 60 per cent of the votes and 64 per cent of the seats, the second 40 per cent of the votes and 36 per cent of the seats. In the other, there are eight parties: four win 15 per cent of the votes and 16 per cent of the seats, while the other four each win 10 per cent of the votes and 9 per cent of the seats. According to the Loosemore-Hanby index, these two outcomes are equally disproportional, and the index returns a value of 4 in each case. Although Lijphart (1985: 10) criticizes the Loosemore-Hanby index for being 'much too sensitive' to the number of parties competing, it might be more appropriate to see it as being much too insensitive. The reasonable assumption Rae's measure attempts to embody is that the first outcome, where each party's vote–seat difference is 4, is less proportional than the second, where each party's difference is just 1.

How can we incorporate this line of thought without encountering Rae's problems? Fortunately, a solution is at hand: the method of least squares, widely used both in the natural sciences and in the social sciences (for example, in fitting a least squares regression line to a collection of data). A least squares index would entail squaring the vote–seat difference for each party; adding these values; dividing the sum by 2; and taking its square root (see Table 3). This gives an index which, like Loosemore and Hanby's, measures disproportionality per election rather than per party and runs from 0 to 100, but unlike theirs it registers a few large discrepancies more strongly than a lot of small ones. In the above example, its value is 4 in the two-party case and 2 in the eight-party case. When there are just two parties, the least squares index takes the same value as the Loosemore-Hanby and Rae indices; otherwise, its value falls between those of the other two. The least squares index can be seen as a happy medium between the Loosemore-Hanby and Rae indices.

It is superior to the 'adjusted Loosemore-Hanby' index discussed by Lijphart (1985:10), which, like Rae's index, measures the amount of disproportionality per party rather than the total amount of disproportionality created at an election. That index differs from Rae's in that it divides the total amount of disproportionality by the 'effective number' of parties (Laakso and Taagepera, 1979), rather than the actual number. It is an improvement on Rae's index, but is more complicated to calculate than the least squares index, and, like Rae's, over-compensates for the number of parties involved. When the votes are split evenly between the contending parties, it returns the same value as the Rae index. It does not have the property of the least squares index of 'penalizing' a few large disproportionalities more than a host of small ones.

Sainte-Laguë Index

The second set of indices focuses not on the absolute differences between seats and votes for each party but on the seats to votes ratio for each party, just as highest average methods do. The relationship between highest average methods and ratio-based measures of disproportionality can be illustrated by considering the derivation of Sainte-Laguë's method, outlined in his short but important 1910 article (for which see Lijphart and Gibberd, 1977: 241–2). The conception of disproportionality that Sainte-Laguë sought to minimize is defined in the following way. For each party, first calculate the amount by which its seats:votes ratio (s/v) differs from the ratio of total seats to total votes (TS/TV); then square this difference and weight the resulting square by the size of the party (v). The error term for each party is thus $v(s/v - TS/TV)^2$. If TS and TV are both expressed in the same terms (for example, as percentages), the error term for each party equals $v(s/v - 1)^2$, or $(s - v)^2/v$. The index, then, involves simply adding $(s - v)^2/v$ for each party (see Table 3).

This gives a measure whose minimum value is zero (when there is full proportionality) and whose maximum value is infinity (when any party with no votes somehow wins a seat). The open-ended nature of its range means that the value of the index in any specific case is, perhaps, less easily interpreted than the Loosemore-Hanby or least squares indices. For example, a value of 6.2 has a clear position on a scale running from 0 to 100 that it does not have on a scale running from 0 to infinity. On the other hand, the Sainte-Laguë index has the major merit of not being subject to paradoxes.

These indices differ fundamentally in that the Loosemore-Hanby, Rae and least squares indices are concerned with the *absolute* difference between a party's shares of the seats and votes, and the Sainte-Laguë index with the *relative* difference. According to the first three indices, there is exactly the same amount of disproportionality involved in a party with 50.1 per cent of the votes winning 55 per cent of the seats and a party with 0.1 per cent of the votes winning 5 per cent of the seats. But according to the Sainte-Laguë index—and, perhaps, intuitively to most people—the former party is only slightly over-represented, with an index of representation of 1.1, while the latter is grossly over-represented, with an index of

50. The Loosemore-Hanby, Rae and least squares indices are not particularly disturbed even by a party with no votes winning a seat, while to the Sainte-Laguë index this is the most disproportional outcome of all.

It would be straightforward to construct an index based on the modified Sainte-Laguë method. This is identical to the Sainte-Laguë index, that is, it involves adding the error terms $(s - v)^2/v$ for each party, except that for parties whose vote share is less than one divided by the total number of seats, $v/(1.4)$ is used instead of v. But this would hardly be a contender as a true measure of disproportionality, since the modification to the Sainte-Laguë formula is made unashamedly to discriminate against small parties.

Equal Proportions Index

The equal proportions method employs a slightly different measure. Huntington (1921: 865) gives an error term for each voter, from which can be derived the error term for each party, namely $\sqrt{(v^2/s - 2v + s)}$, assuming *TS* and *TV* have the same value. The overall error term is thus computed by calculating v_i^2/s_i for each party; adding these terms; subtracting 100 (if seats and votes are both expressed as percentages); and taking the square root of the result. The uselessness of this for practical purposes is at once apparent: any party with no seats has an error term infinitely large, so minimizing the sum of the error terms necessitates giving every party at least one seat. This follows logically from the nature of the equal proportions method (see above), but renders the measure valueless for our needs.

D'Hondt Index

There is no d'Hondt index to be devised by summing any 'error terms' for each party. The d'Hondt method does not work by trying to minimize some overall measure of disproportionality. It is not true to say, as do Lijphart and Gibberd (1977: 235), that the d'Hondt formula seeks to minimize the sum of the highest unrewarded averages—that is, the terms $v_i/(1 + s_i)$—for each party. Compare, for example, the award of the second seat when party *A* has 60 votes and party *B* 28 votes, as in Table 1. According to Lijphart and Gibberd it should go to party *B*, as the highest unrewarded averages of the two parties, 30 and 14, would then add to 44, compared with 20 and 28 (adding to 48) if it goes to *A*. But in practice it will go to *A*, because *A*'s second average of 30 exceeds *B*'s first average of 28.

The d'Hondt method, as we noted at the start of the paper (p.34 above) has only one over-riding aim, namely to keep to a minimum the over-representation of the most over-represented party. Consequently, if there was to be a d'Hondt index, it would have to be simply this: the seats to votes ratio of the most over-represented party (see Table 3). The minimum value of this index is 1 (when all parties have an identical seats:votes ratio), and the maximum, attained if a party with no votes somehow wins a seat, is infinity. The disadvantage of such an index is that it is vulnerable to producing a freak score if a small party gains some degree of over-representation. For example, at Italy's 1983 general election, the tiny Val d'Aosta Union won 0.159 per cent of the seats with 0.076 per cent of the votes, achieving an index of representation of 2.085, which makes this election the most disproportional in the entire data set analysed below according to this index.

Even though it could be argued that this is in keeping with the spirit of the d'Hondt method, which does indeed take a very dim view of the over-representation of small parties, it makes the index inherently unsatisfactory, as the 1983 Italian election might have been highly proportional by every other criterion; it might be that another small party with 0.083 per cent of the votes won no seats, and that every other party was represented with perfect accuracy. A sensible amendment would be to take the seats to votes ratio of the most over-represented party that wins 5 per cent of the votes or more. In most cases the party in question is the largest one, and there would be little point in trying to refine the index further by raising the cutoff point to, say, 10 per cent.

The d'Hondt index's reliance on the seats:votes ratio for just one party rather than the sum of the disproportionalities of each party makes it similar in some ways to another measure sometimes employed, namely the share of the votes below which parties tend to be under-represented. This has been used by Taagepera and Laakso (1980) in constructing 'proportionality profiles' of the results produced by electoral systems, and by Katz (1984: 136) as a measure of proportionality. The measure may in practice correlate well with other notions of proportionality—in plurality systems, the break-even point is far higher than in PR systems—but it is only an indirect measure, as the break-even point does not of itself tell us how far parties on either side of the point tend to be under- (or over-) represented.

Additional Factors Influencing Disproportionality

Before proceeding to apply these indices to actual election results, there is an important point to bear in mind. It would be wrong to assume that all, or even most, of the disproportionality (however measured) that we observe at an election held under a particular electoral system can be attributed to the specific seat allocation formula used. There are two, and sometimes four, other main sources of disproportionality: first, the distribution of votes between the parties; second, the impact of district magnitude; third, the possibility of malapportionment; and fourth, the use of thresholds.

If we consider the contest illustrated in Tables 1 to 3, it is apparent that the absence of perfect proportionality there is due in the first instance to the fact that it is simply impossible to divide five seats among parties whose votes split 60-28-12 without creating some disproportionality. The Loosemore-Hanby index has a value of 8 when applied to the outcome in Table 2, but this disproportionality has not been created by the largest remainders method. On the contrary, since the Loosemore-Hanby index mirrors the decisions of the largest remainders method, none of the disproportionality registered by this index can be attributed to the method. This might suggest that it would be fairer to measure disproportionality not as the difference between the actual outcome and perfect proportionality but as the difference between the actual outcome and the highest degree of proportionality that was attainable under the circumstances. However, although this idea might be worth pursuing in principle (for some discussion, see Irvine, 1988), it must be borne in mind that even the single-member plurality system, as used in Britain and the United States, delivers the maximum degree of proportionality attainable (by any measure) within each constituency.

The second source of disproportionality is the important variable of district magnitude (the number of members returned per constituency), whose full impact we do not have space to assess in this paper. It is undoubtedly a major determinant of the proportionality of any election outcome; under a PR system, proportionality will, other things being equal, vary directly with district magnitude. The impact of district magnitude upon proportionality was first studied by Rae (1971: 114–25), whose analysis has subsequently been refined by Lijphart (1990: 486–91). There is strong evidence, indeed, that district magnitude affects proportionality more than the electoral formula does (Taagepera and Laakso, 1980: 443; Taagepera and Shugart, 1989: 112–25).

Certainly, when district magnitude equals 1, all formulae (apart from those based on eliminations and runoffs) produce the same outcome. Likewise, as district magnitude approaches infinity, so the outcome produced by every PR formula approaches perfect proportionality. Formula matters only within a certain range of district magnitude. It is hard to be more precise, as the upper limit of the range varies according to the distribution of votes between parties. The fewer the number of competing parties, the smaller is the district magnitude needed to achieve a given level of proportionality. In the case of the context illustrated above in Tables 1 to 3, the d'Hondt, modified Sainte-Laguë and largest remainders formulae produced different outcomes in the five-seat constituency, but if there had been ten seats at stake, each of these formulae, and the pure Sainte-Laguë method, would have produced the same 6-3-1 outcome.

In practice, district magnitude varies quite a bit between countries. Our data are drawn from the 23 countries (excluding the United States) covered by Mackie and Rose (1982); details of district magnitude are taken from Mackie and Rose (1982: 410–11) and Mackie and Rose (1991: 509–10). Five of the 23 countries have used single-member constituencies: Australia, Canada, France (in 1981 and 1988), New Zealand and the United Kingdom. Five have employed a PR formula in relatively small constituencies (average district magnitude below 10) with no higher tier allocations to iron out discrepancies arising from the constituency results: France (in 1986), Ireland, Japan (whose single non-transferable vote system is in any case best regarded as semi-PR), Norway (in 1981 and 1985) and Spain. Six use PR in large constituencies (average district magnitude ten or above) with no higher tier allocations: Finland, Israel, Luxembourg, the Netherlands, Portugal and Switzerland (in Israel and the Netherlands the only allocation is at the national level, so it might more sensibly be said that there is no lower tier). The other nine cases all have both a lower and a higher tier: Austria, Belgium, Denmark, West Germany, Greece, Iceland, Italy, Norway (in 1989) and Sweden. Among these nine, some reserve a specific proportion of seats in advance for higher tier allocation: this proportion ranges from 4.8 per cent (8 out of 165) in Norway in 1989 up to 50 per cent (West Germany).

In all but the last nine cases, plus Israel and the Netherlands, the efficacy of the PR formula is at the mercy of the outcome in the constituencies. Whether the overall outcome is highly proportional may depend much more on whether the disproportionalities within each constituency largely even themselves out across the country or are cumulative. Either way, the electoral system should not take the credit or the blame, and any test based on the results aggregated for the whole country runs the risk of committing the ecological fallacy. Ideally, a test for the proportionality of an electoral formula should concentrate on the average amount of disproportionality produced within each constituency rather than the outcome at the national level. However, for the moment the data needed to proceed along these lines are not available.

A third source of disproportionality can arise if some areas of a country are allocated more seats in relation to population than are others. This factor has not been systematically studied, but it has been significant in some countries, usually involving the deliberate over-representation of rural areas. It has to be taken into account when seeking explanations for the consistent over-representation of the National Party in Australia, the Liberal Democratic Party in Japan and the Progressive Party in Iceland.

Fourth, the use of thresholds (significant in Denmark, West Germany, Greece in 1981 and 1985, Iceland in 1987, Norway in 1989 and Sweden, and also employed in Austria, Greece in 1989, Iceland in 1979 and 1983, Israel and the Netherlands) is in some cases another obstacle to the maximization of proportionality.

The Indices Compared: A Practical Application

We now turn to an empirical test of the six indices discussed above, based on parliamentary elections from 1979 to 1989 inclusive in 23 of the 24 countries (US presidential elections being excluded) covered in Mackie and Rose (1982). The data are taken from Mackie and Rose (1982) and from Mackie's annual updates in the *European Journal of Political Research*, cross-checked against the results provided regularly in *Electoral Studies* and *West European Politics*. This gives a data set of 82 elections.

Although there are only 23 countries, we have 26 cases, because of significant changes to the electoral system in three countries (France, Greece and Norway) during this period (see note to Table 4). Two other changes we do not regard as significant. The first is the change in West Germany, from 1984 onwards, from d'Hondt to largest remainders to allocate the national seats; the large district magnitude here renders the formula of very little relevance. In 1980, when the d'Hondt formula was used, a change to largest remainders would have altered the allocation of just one of the 497 seats (which would have gone from the CDU/CSU to the FDP); in 1983 it would have affected two seats; and in 1987, if the change to largest remainders had not been made, again two seats would have been affected. This emphasizes the point that district magnitude in West Germany is better seen as 496 (the total number of seats at stake, barring *überhangmandate*) rather than somewhere in the 5–10 range, as suggested by Lijphart (1990: 489). The second change that we regard as minor is the switch from d'Hondt to largest remainders as a method of allocating the lower tier seats in Iceland from the 1987 election onwards. Given that the national seats (13 out of the total of 63 in 1987) are still allocated by d'Hondt, in such a way as to make the overall allocation as proportional as possible (where proportionality is seen from the perspective of the d'Hondt method), the change in the formula used to allocate the constituency seats is unlikely to make much difference. The effect of the d'Hondt method at the higher level is to over-ride the impact of largest remainders at the lower level. Our expectation that this change will have little effect on overall proportionality, however measured, is confirmed by the evidence.

The average values for each country on each of the six indices are given in Table 4, with the matrix of correlations between the indices shown in Table 5. In general terms, the indices give a similar picture, with the countries using plurality or majority formulae (Australia, Canada, France (in 1981 and 1988), New Zealand and the UK) near the bottom on every measure. The other countries consistently near

TABLE 4. Disproportionality at 82 elections in 23 countries 1979–89, as measured by six indices

	Least squares	Loosemore-Hanby	Rae	Sainte-Laguë	D'Hondt	D'Hondt (5%)
West Germany (3)	1.0	1.4	0.5	1.3	1.02	1.02
Netherlands (4)	1.4	2.7	0.4	2.1	1.08	1.04
Austria (3)	1.5	2.3	0.8	2.3	1.12	1.02
Denmark (5)	1.8	3.4	0.6	3.5	1.09	1.06
Sweden (4)	1.9	2.9	0.8	3.0	1.04	1.04
Italy (3)	2.7	4.8	0.7	3.0	1.81	1.08
Iceland (3)	2.8	4.2	1.3	2.3	1.23	1.16
Ireland (5)	3.3	4.7	1.4	2.9	1.15	1.09
Israel (3)	3.3	5.7	0.6	5.0	1.12	1.08
Finland (3)	3.3	6.5	1.3	4.4	1.20	1.11
Switzerland (3)	3.5	6.8	0.8	5.4	1.50	1.12
Norway B (1)	3.8	4.8	1.1	4.8	1.11	1.11
Belgium (3)	3.9	8.2	1.1	6.9	1.26	1.26
Greece B (2)	4.2	5.8	2.0	3.2	1.08	1.08
Portugal (5)	4.3	6.6	1.6	5.6	1.12	1.12
Luxembourg (3)	4.4	7.7	1.9	5.8	1.13	1.13
Norway A (2)	4.8	8.6	2.0	7.1	1.18	1.18
Japan (4)	5.7	8.3	2.3	4.6	1.15	1.15
France B (1)	7.4	12.2	3.2	8.5	1.17	1.17
Greece A (2)	7.7	10.3	3.5	9.3	1.18	1.18
Australia (4)	9.4	13.2	5.9	12.8	1.47	1.47
Spain (4)	9.7	15.4	2.2	14.7	1.27	1.27
Canada (4)	13.0	16.0	5.8	14.4	1.34	1.34
New Zealand (3)	14.0	17.1	9.2	19.4	1.32	1.32
France A (2)	14.3	19.6	5.9	20.1	1.43	1.43
United Kingdom (3)	16.6	20.0	7.0	23.5	1.49	1.34
Average value	5.6	8.1	2.4	7.3	1.23	1.16

1. Figures in brackets denote the number of elections for each case during the period covered.
2. France A: elections of 1981 and 1988 (double-ballot system). France B: election of 1986 (d'Hondt in small constituencies with no higher level tier of allocation).
3. Greece A: elections of 1981 and 1985 ('reinforced' PR, incorporating high thresholds for qualification for higher tier seats). Greece B: elections of June and November 1989 (thresholds for entitlement to share in higher tier seats greatly reduced).
4. Norway A: elections of 1981 and 1985 (d'Hondt with no higher tier of allocation). Norway B: election of 1989, when a higher tier of eight seats was introduced to reduce disproportionalities in the constituencies, which continued to have 157 seats.

TABLE 5. Correlations between values of indices of disproportionality at 82 general elections 1979–89

	Loosemore-Hanby	Rae	Sainte-Laguë	D'Hondt	D'Hondt (5%)
Least squares	.98	.91	.97	.48	.81
Loosemore-Hanby		.87	.97	.51	.83
Rae			.88	.42	77
Sainte-Laguë				.50	.81
D'Hondt					.59

Note: The values reported are Pearson's correlation coefficient (r). Every coefficient in the table is significant at the .001 level.

the bottom are France (in 1986), Greece (1981 and 1985) and Spain. France in 1986 and Spain both employed PR with the d'Hondt formula in small constituencies with no higher tier, and this emerges as little more proportional than Greece's notorious 'reinforced' PR.

But the six indices do not give identical rankings. The clear outlier is the unmodified d'Hondt index, which correlates relatively weakly with the other five indices (Table 5) and produces some highly deviant rankings. Italian elections emerge as the least proportional, due entirely to the consistent success of the tiny Val d'Aosta Union in the single-member Val d'Aosta constituency, whereas Italy is in the top third according to every other index. Switzerland, ranked in the middle by the other indices, emerges as the next most disproportional. The unmodified d'Hondt index is unduly sensitive to small shifts in the support for very small parties, which trigger their gain or loss of a seat.

The other two ratio-based indices produce rankings much closer to the mainstream, though like all ratio measures their values can be affected significantly by small changes in the fortunes of one party. The d'Hondt (5 per cent) index may also be affected by whether a party reaches the cutoff point or not. For example, it, like the other indices, ranks Austrian elections as among the most proportional, but if the FPÖ had won 5 per cent of the votes in 1983 instead of 4.98 per cent, Austrian elections would have been only the eleventh most proportional on this index. Indices that employ an arbitrarily-determined cutoff point may be seen as inherently unsatisfactory.

The Sainte-Laguë index correlates more strongly with the difference-based indices than do the d'Hondt indices (Table 5), and at the theoretical level is probably the soundest of all the measures (pages 41–2 above). If it has a drawback, it is that it can be affected by the amount of information available on the fate of small groups. In many cases, the largest single contributor to disproportionality as measured by the Sainte-Laguë index comes from the small parties and 'Others' who win no seats. If every group for which a vote total is reported wins a seat, the result looks much more proportional. The two most proportional results in the whole data set, according to the Sainte-Laguë index, were those of the elections in Iceland in 1979 and Ireland in February 1982, when no party or group was left seatless. While this is valid in terms of the construction of the index, it also means that it can be unhealthily dependent on exactly how the votes and seats for micro-party and independent candidates are aggregated and reported.

Of the three difference-based indices, Rae's stands a little apart from the other two, correlating less strongly with them than does the Sainte-Laguë index. It is too dependent on the number of parties standing. It rates Israel as the fourth most proportional case, in contrast to Israel's upper mid-table (9th to 13th) ranking on the least squares, Loosemore-Hanby and Sainte-Laguë indices, because it divides Israel's quite sizeable amount of total disproportionality by such a large number of parties (13 in 1981, 15 in both 1984 and 1988). Similarly, Spanish elections, which produce a higher than average amount of disproportionality according to every other index, produce slightly less than average as measured by the Rae index, again because of the large number of parties in Spain (12 in 1979, 10 in 1982, 12 in 1986 and 13 in 1989). In contrast, Austria is ranked only eighth, whereas it is in the top three on four of the other five indices, because Austria's very small amount of total disproportionality is divided by only a small number of parties (3 in 1979, 6 in 1983 and 5 in 1986). Practical application bears out the theoretical doubts about the soundness of Rae's measure.

The remaining two indices, least squares and Loosemore-Hanby, correlate very strongly with each other (Table 5). The ranking orders the two measures produce are very similar: seven cases have identical rankings and another eleven differ by only one place, with five being two places apart and the other three cases three places apart. Where the two rankings for a country differ by more than one position, the least squares ranking tends to fall between the Rae and the Loosemore-Hanby rankings, bearing out the theoretical justification for this measure (see *Rae and Least Squares Indices* above). The least squares index does pay some heed to the number of parties involved and the votes–seats difference of each, whereas Loosemore-Hanby pays none and Rae pays too much. The least squares index, like that of Sainte-Laguë, can be affected by the extent of information available on the fortunes of those parties that win no seats. If the votes of unrepresented parties are lumped together in one block, then the least squares index registers greater disproportionality than if each is treated separately, precisely because the index distinguishes between cases where there are a few large vote–seat disparities and cases where there are many small ones. Provided that the votes for unrepresented parties are sufficiently disaggregated in the data available, the slight extra degree of sensitivity shown by the least squares index to the way in which a given total amount of disproportionality is produced tips the balance in its favour over the Loosemore-Hanby index.

The caveats noted above (*Additional Factors Influencing Disproportionality*) are borne out by the rank ordering of the countries in Table 4. If these other factors were not operating, we should expect countries using largest remainders to emerge at the top according to the Loosemore-Hanby index, countries using d'Hondt to rank highest on the d'Hondt index, and so on. This does not happen; the country with the most proportional elections, according to Loosemore-Hanby, used d'Hondt for two of the three elections covered (and would have headed the rankings with an identical score on the basis of these two elections), while some elections held under d'Hondt, as in Spain and Switzerland, are rated well down the list according to the d'Hondt index.

Instead, district magnitude emerges clearly as the main determinant of a country's ranking. The top seven countries (according to the least squares index) all have higher tier allocations to overcome disproportionalities arising at the constituency level. (The Netherlands, which is one large constituency, has only a higher tier.) There are only four other countries which use higher tier allocations. One is Israel, which has essentially the same electoral system as the Netherlands; its results are less proportional partly because of its slightly higher exclusion threshold (1 per cent compared with 0.67 per cent) and partly because of the much greater fragmentation of the vote and the larger number of votes wasted on unelected 'Others'. A second is Belgium, where the higher tier constituencies (the nine provinces) are too small and numerous to offset the disproportionalities arising in the 30 lower-tier constituencies. The third is Norway in 1989, where the very small number of additional seats was not sufficient to overcome the disproportionality—in particular, the considerable overrepresentation of Labour—created in the constituencies. However, it is worth noting that the eight higher tier seats introduced in 1989 made a clear difference. If they had not been created, the 1989 Norwegian election would have been virtually as disproportional as those of 1981 and 1985, and the higher ranking of Norway B compared with Norway A in Table

4 is due almost entirely to the change in the electoral system (details of the results and the allocation of the additional seats are taken from Aardal, 1990). The fourth case is Greece in 1981 and 1985, where the exceptionally high thresholds (17 per cent of the votes) that had to be attained to qualify for higher tier seats made the latter a source of greater rather than less disproportionality. Overall, then, this analysis bears out the conclusions noted earlier to the effect that district magnitude is a more important determinant of proportionality than is electoral formula.

The first factor mentioned in *Additional Factors* ... (above), the distribution of votes between the parties within each constituency, and whether disproportionalities within constituencies tend to cumulate or to even themselves out across the country, helps to explain much of the remaining variance. It must explain the high ranking of Ireland, despite its small average district magnitude (only 4 seats per constituency). It is true that the indices we employ here measure only the relationship between a party's share of the seats and its share of the first preference votes, ignoring the impact of vote transfers, which, under STV, will affect the final allocation. However, there is no reason to believe that this either underestimates or exaggerates the proportionality of Irish election results (Gallagher, 1986: 255–7). And, leaving aside the possibility of malapportionment, it is the only variable that can explain why Luxembourg's elections are rather more disproportional than those of Finland and Switzerland, according to most indices, even though the three countries employ virtually identical electoral systems as far as the awarding of seats to parties is concerned, all using the d'Hondt method with no higher tier seats. District magnitude is in a similar range in each country, and in any case Luxembourg's average district magnitude is the highest of the three (15 compared with 14 in Finland and 10 in Switzerland).

The distribution of votes between the parties also explains why Germany tops the list despite its 5 per cent threshold. Paradoxically, a threshold of this size may have a less adverse effect on proportionality than a lower one, as small parties become victims of the 'wasted vote' syndrome just as they do under the plurality system. In consequence, very few votes (an average of 1.3 per cent) were cast at the three German elections of our period for parties that did not attain the threshold. Lower thresholds, as in Sweden and Israel, encourage more mini-parties to stand and more voters to support them, so that more votes (an average of 2.9 per cent in Sweden and 3.5 per cent in Israel) are cast for parties that fail to reach the threshold.

Conclusion

PR methods all set out to minimize disproportionality. They differ from each other not because some try harder than others, or because some are 'fairer' than others, but because they embody different ideas as to how disproportionality should be measured. Each method of PR minimizes disproportionality according to the way it defines disproportionality, and thus each in effect generates its own measure of disproportionality. Of the various measures based on the largest remainders method, the least squares index seems to have advantages over the more widely used Loosemore-Hanby and Rae indices. Among the indices based on highest average methods, Sainte-Laguë's has clear merits when compared with indices based on the d'Hondt method. Its invulnerability to the paradoxes to which the measures based on largest remainders are susceptible is a strong argument in favour of its adoption as the standard measure of disproportionality.

Applying the various methods to the results of recent general elections suggests that there can be considerable variation between the rankings they produce. The Rae index is too sensitive to the number of parties competing, while the d'Hondt index can be too dependent on the fortunes of just one party. The results produced by the least squares, Loosemore-Hanby and Sainte-Laguë indices correlate very strongly with each other.

The ranking order each index produces suggests that district magnitude is a more important determinant of proportionality than formula. Formula matters only within a certain range of district magnitude. When district magnitude is large, either because constituencies are large or because of the use of higher tier seat allocation, the proportionality of election outcomes is high, regardless of the formula or the index of measurement employed.

Appendix

Proof that the Loosemore-Hanby index of disproportionality is by definition minimized by the largest remainders seat allocation

The largest remainders formula works as follows. First, each party wins a seat for each Hare quota in its vote total. This allocation produces no disproportionality at all as between seats awarded and Hare quotas used up, since the Hare quota is calculated as total votes divided by total seats.

Then the remaining seats are awarded to parties according to which have the largest number of unused votes. The Loosemore-Hanby index will then be calculated as the sum of the under-representations of the under-represented parties, or the sum of $(r_1 + r_2 + \ldots r_n)$, where r_i is the remainder of the ith party not awarded an additional seat and r_n is the largest of these unrewarded remainders. In the case of the example in Table 2, the only such remainder is that of party *B* (8,000). This sum, of course, has exactly the same value as the sum of the over-representations of the over-represented parties, the sum of $((q_1 - a_1) + (q_2 - a_2) + \ldots (q_m - a_m))$, where a_j is the rewarded remainder of the jth party to be awarded an additional seat (a_1 being the smallest of these rewarded remainders), and q_j equals the product of the Hare quota and the number of seats awarded to the jth party. In Table 2, the sum of these over-representations is simply the over-representation of party *C*: 20,000 minus 12,000. Since the formula rewards the largest remainders, it follows that $a_1 \geqslant r_n$. Consequently, the largest remainders formula, in minimizing the sum of $(r_1 + r_2 + \ldots r_n)$, also by definition minimizes the value of the Loosemore-Hanby index.

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